Computability of Data Word Functions Defined by Transducers

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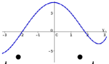
Last Year at Highlights. . .



Computability and Continuity of Regular Functions

Work by Vrunda Dave, Emmanuel Filiot, S. N. Krishna and ***Nathan Lhote***

HIGHLIGHTS 2019 WARSAW

Motivation: Synthesis

Input: A specification $S \subseteq I^\omega \times O^\omega$

Output: A deterministic machine whose behaviours satisfy S
i.e. computing $f : I^\omega \to O^\omega$ such that $f \subseteq S$

Computability

Hypothesis: S is already a function

$$f : I^{\omega} \to O^{\omega}$$

Input: A *functional* specification f

Output: A deterministic program computing f

Computability for non-terminating behaviours

f is *computable* if there exists a deterministic Turing machine M with:

- A 1-way read-only input tape
- A 2-way read/write working tape
- A 1-way write-only output tape

➔ On reading longer and longer prefixes of the input w
M produces longer and longer prefixes of the output $f(w)$

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Examples

- $u\sigma\#^\omega \mapsto \sigma u\#^\omega$
- $u \mapsto p_1 p_2 \ldots$ where $p_i = 1$ iff i is prime

Counter-examples

- $d_1 d_2 \ldots \mapsto \begin{cases} d_1^\omega \text{ if } d_1 \text{ repeats} \\ d_2^\omega \text{ otherwise} \end{cases}$
- $u \mapsto h_1 h_2 \ldots$, where $h_i = 1$ iff the i-th Turing machine halts

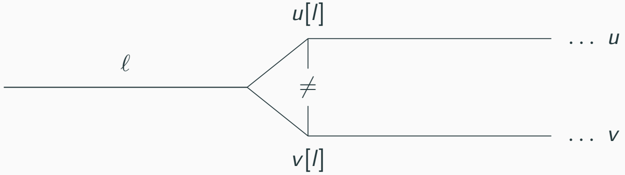
Continuity

Definition

➔ The one you know: if two inputs are close, then their outputs should be close as well.

Cantor Distance

➔ Two words are close if they coincide on a long prefix.



$d(u, v) = 2^{-\|\ell\|}$

Continuity

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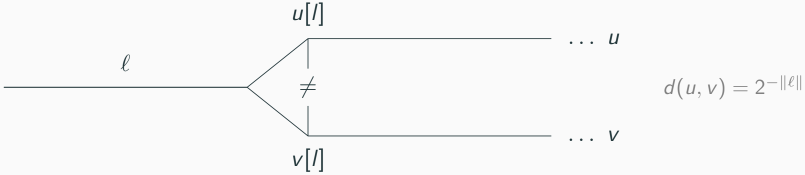


Theorem [Dave et al., 2020]

For *regular functions*, computability and continuity coincide and are decidable in polynomial time.

Extension to the Realm of Data Words

Data Words

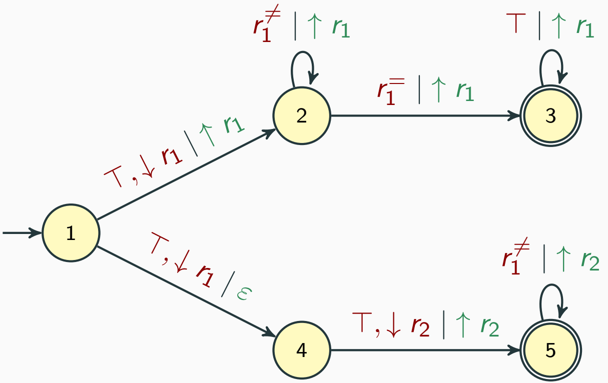
Infinite words over an infinite alphabet $\mathcal{D}$, with some structure, e.g. $\mathcal{D}$ is $(\mathbb{N}, =)$, $(\mathbb{Q}, \leq)$ or $(\mathbb{N}, \leq)$

Nondeterministic Register Transducer

Finite 1-way transducer + finitely many registers

- ➔ Store input data
- ➔ Compare w.r.t. the structure
- ➔ Output data

Example of a NRT



A NRT computing $f_{\text{repeat}} : d_1 d_2 \ldots \mapsto \begin{cases} d_1^{\omega} & \text{if } d_1 \text{ repeats} \\ d_2^{\omega} & \text{otherwise} \end{cases}$

Results

Main Result

For functions *defined by* NRT, computability and continuity again coincide and are decidable and $\textsc{PSpace}$-complete when $\mathcal{D}$ is $(\mathbb{N}, =)$ (and $(\mathbb{Q}, \leq)$)

Also:

- Decidability of functionality for NRT
- Closure under composition
- Polynomial-time subclass: test-free NRT

Conclusion

Take-home Message

- Computability = Continuity for functions defined by nondeterministic register transducers over $(\mathbb{N}, =)$
- Nice proof techniques

Ongoing Work

- $(\mathbb{Q}, \leq)$ (and oligomorphic domains)
- Nondeterministic reassignment

➔ Paper, slides, poster and 30mn video: `shorturl.at/jxDIJ`