History-Deterministic Register Automata

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Friday, November 5th, 2021

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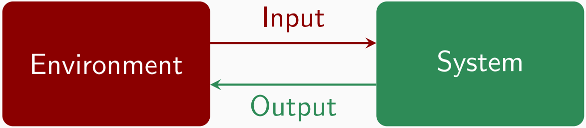
Motivation: Reactive Synthesis

Reactive systems



Interaction $\rightsquigarrow$ $i_1 o_1 i_2 o_2 i_3 o_3 \ldots$

Goal

Generate a system from a specification

$? \parallel \mathrm{Env} \models \mathrm{Specification}$

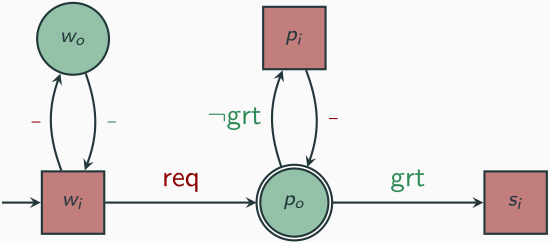
The Finite Alphabet Case

Specification = ω-regular language, as an MSO or LTL formula.

Example

Any request is eventually granted:

$$G(req \Rightarrow F(grt))$$



A non-deterministic Büchi automaton checking that some request is left unsatisfied

Theorem [J.R. Büchi and L.H. Landweber, 1969]

The reactive synthesis problem is decidable for ω-regular specifications.

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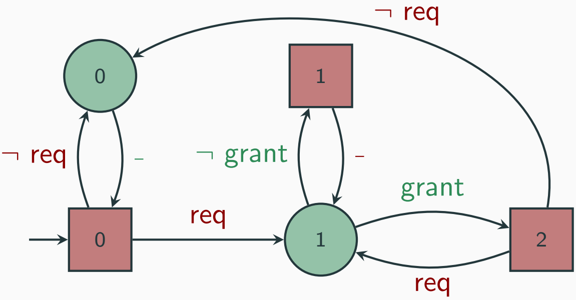
- Specification $\rightsquigarrow$ equivalent deterministic automaton A
- Resolve an infinite duration game played over A



A parity game corresponding to $G(\text{req} \Rightarrow F(\text{grt}))$.

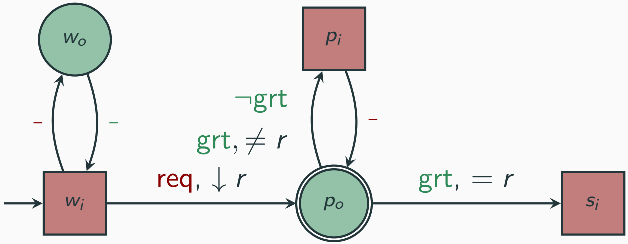
Lifting to Infinite Alphabets: Register Automata [Kaminski and Francez, 1994]

➔ Requests are specifically granted to a client

Register Automata

Finite automata with a finite set R of registers

- Store data
- Test register content



A non-deterministic Büchi *register* automaton checking that some request is left unsatisfied

Lifting to Infinite Alphabets: Register Automata [Kaminski and Francez, 1994]

Theorem [Exibard et al., 2021]

The synthesis problem is decidable for specifications given as deterministic register automata.

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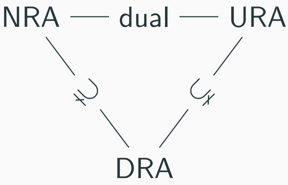
Problem

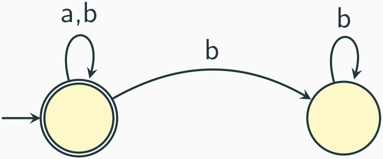
Register automata do not determinise!

➔ “Some two data values are identical”



History-Determinism [Henzinger and Piterman, 2006] [Colcombet, 2009]

"Non-deterministic choices do not depend on the future"



A non-deterministic co-Büchi automaton which is not history-deterministic

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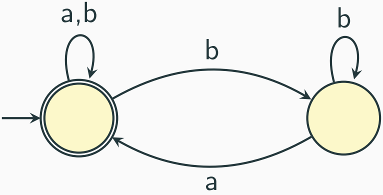

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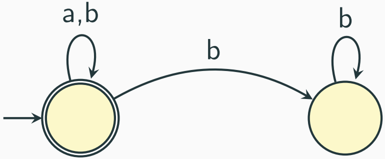

A non-deterministic co-Büchi automaton which is not history-deterministic

The Letter Game

- Two-player game
- Adam chooses a letter
- Eve picks a transition of the automaton
- She wins iff either $\begin{cases} \text{she built an accepting run over } w \\ \text{Adam gave a word } w \notin L(A) \end{cases}$
- Winning strategy: $\lambda : \Sigma^+ \to \Delta$

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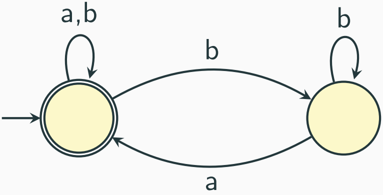

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History-Determinism [Henzinger and Piterman, 2006]

Succinctness [Kuperberg and Skrzypczak, 2015]

History-deterministic co-Büchi automata can be *exponentially* more succinct than deterministic ones.

Good-for-gameness [Henzinger and Piterman, 2006]

History-deterministic ω-regular automata "compose well" with games.

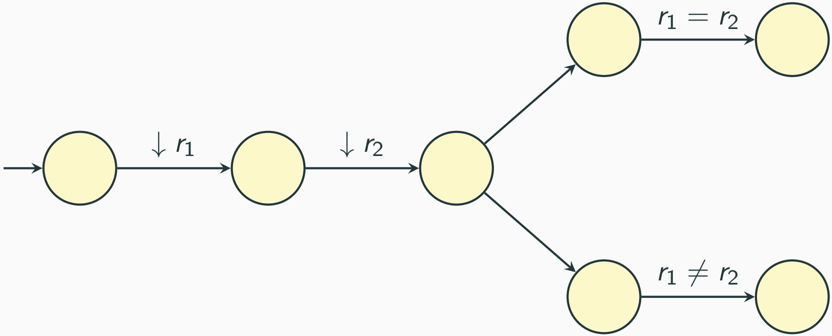
➔ For any game G, the winner of G with winning condition $L(A)$ is the same as the winner of $G \otimes A$.

Lifting to Infinite Alphabets: History-Deterministic Register Automata

Expressivity and Succinctness

Over finite words, history-determinism are as expressive, but exponentially more succinct.

➔ Resolvers depend on equalities between registers.

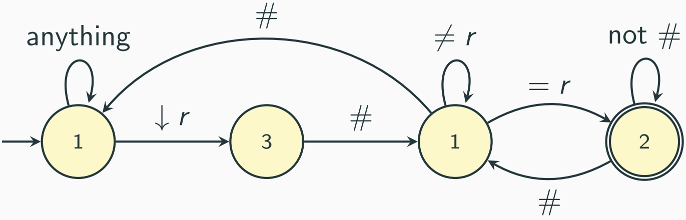


Expressivity and Succinctness

Over *infinite* words, history-determinism is strictly more expressive than determinism.

➔ Blocks of data such that eventually,
some data appears in all blocks

$$L = \left\{ d_0^0 d_1^0 \dots d_{n_0}^0 \# d_0^1 d_1^1 \dots d_{n_1}^1 \# \dots \;\middle|\; \begin{array}{l} \exists d \in \mathcal{D}, \exists N \geq 0, \\ \forall i \geq N, \exists 0 \leq j \leq n_i, d_j^i = d \end{array} \right\}$$



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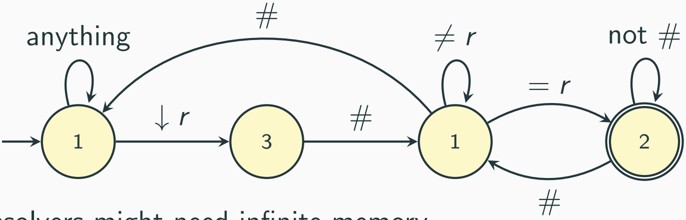

➔ Resolvers might need infinite memory

History-Deterministic Register Automata

Equivalence with Good-for-Gameness

- Recall that over ω-regular languages, history-determinism and good-for-gameness coincide

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Decidability

Over finite words history-determinism is decidable.

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Decidability

Over finite words history-determinism is decidable.

➔ The letter game is then equivalent with the 1-token game, where Adam also constructs a run.

Conclusion

- Relevant class, with applications to synthesis
- Finite word case: runtime verification $\rightsquigarrow$ monitor synthesis for properties with data
- Better understanding of non-determinism

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Open problems

- Equivalence with GFG-ness over ω-words
- Decidability of the notion over ω-words
- Are games with co-non-deterministic winning conditions
    - Determined?
    - Decidable?
    - What is their memory structure?

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