INFO-F-410 Embedded Systems Design
Control Theory

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1 Introduction

Let us first briefly recall the equations that allow to model a closed loop system with a PID controller. The variables are :

1. r, the reference value. The output should always be as close as possible to r.
2. u_t, the input of the environment that will be connected to the output of the controller.
3. y_t, the output of the system (at time t), i.e. the output of the environment.
4. e_t, the error (at time t), equal to $r - y_t$. It is the input of the controller.
5. w_t, the perturbation (at time t). It is an input to the environment.

The control law of a PID controller is

$$u_{t+1} \quad = \quad P \times e_t + D \times (e_t - e_{t-1}) + I \times \left(\sum_{i=1}^{t} e_i \right)$$

Intuitively, the derivative term $D \times (e_t - e_{t-1})$ allows to keep the oscillations as low as possible, while the integrator term $I \times \left(\sum_{i=1}^{t} e_t \right)$ allows to enforce convergence to the reference value.

2 Exercise 1 : *Cruise Control*

In this first exercise, we'll exploit the example of the course, of a system to regulate the speed of a car. The equation that describes the car's speed is as follows (where y_t is the speed at time t) :

$$y_{t+1} \quad = \quad 0,7 \times v_t + 0,5 \times u_t - w_t$$

- Build a Scade node with inputs u_t and w_t, output y_{t+1} that model this equation. Fix the initial speed to 20.
- Build a Scade node for the PID controller (P, I, and D parameters can be Scade constants).
- Combine these two nodes to build a model of the whole closed loop system.

Now, we have to determine proper values for parameters P, I and D. This is called *PID Tuning…* To start, we fix the disturbance w to 0.

- Assign the value 0 to parameters I and D, and observe, in the Scade simulator, the behaviour of the system for different values of P (by letting $r = 50$, and initial speed $= 20$). Try at least the following values for P : -3, 0.5, 2 and 5.

- Determine a value for P that seems adequate, i.e., yields a fair accuracy (less than 20% error) without letting the system diverge.
- Then, adapt D to lower oscillations, and further adapt I to let the system converge. Remind that $D \leq 0$ and $I \geq 0$.
- How does the system react to disturbances ? Try $w = 5$ and $w = -5$.

An empirical method to tune P, I and D has been introduced by ZIEGLER and NICHOLS in 1942. Apply it and compare to your previous findings :
- First let $I = D = 0$.
- Then, gradually increase P up to the point where the system oscillates and diverges. This value of P is called *ultimate gain*, and is denoted P_u. The oscillation period is called *ultimate period* and is denoted T_u.
- Then, let $P = 0.6 \times P_u$, $I = 0.5 \times T_u$ and $D = 0.125 \times T_u$.
- How does the system react to disturbances ? Try $w = 5$ and $w = -5$.

Once appropriate values for P, I and D have been found, the code of the controller can be automatically generated by Scade, as explained in Section 4 of the *Getting started* guide.
- Generate the code corresponding to the controller node.
- Use this code in a C program that reads on the input a sequence $r_1, r_2, r_3 \ldots$ of reference speeds, and displays the output of the controller, as well as the car's speed. Fix the initial speed at 20, and the disturbance at 0.

To finish, modify the controller to take into account saturation of the u_t variable. The possible values for u_t are in $[0, 45]$. Observe how the response of the system is changed by this.

3 Exercise 2 : thermostat

We will now consider a system in which we have to control a heater. The input r of the controller is the desired temperature. The equation that model the environment is :

$$u_t \quad = \quad 0,7 \times y_t - 0,0035 \times y_t^2 + 0,4 \times u_t$$

Design, thanks to Scade, a PID controller for that system. The reference temperature will be between 0 and 100. Compute the P, I, and D values with thee ZIEGLER–NICHOLS method, considering two different references : 20 and 90, and using 10 as initial temperature.