A Lattice Theory to Solve Games of Imperfect Information

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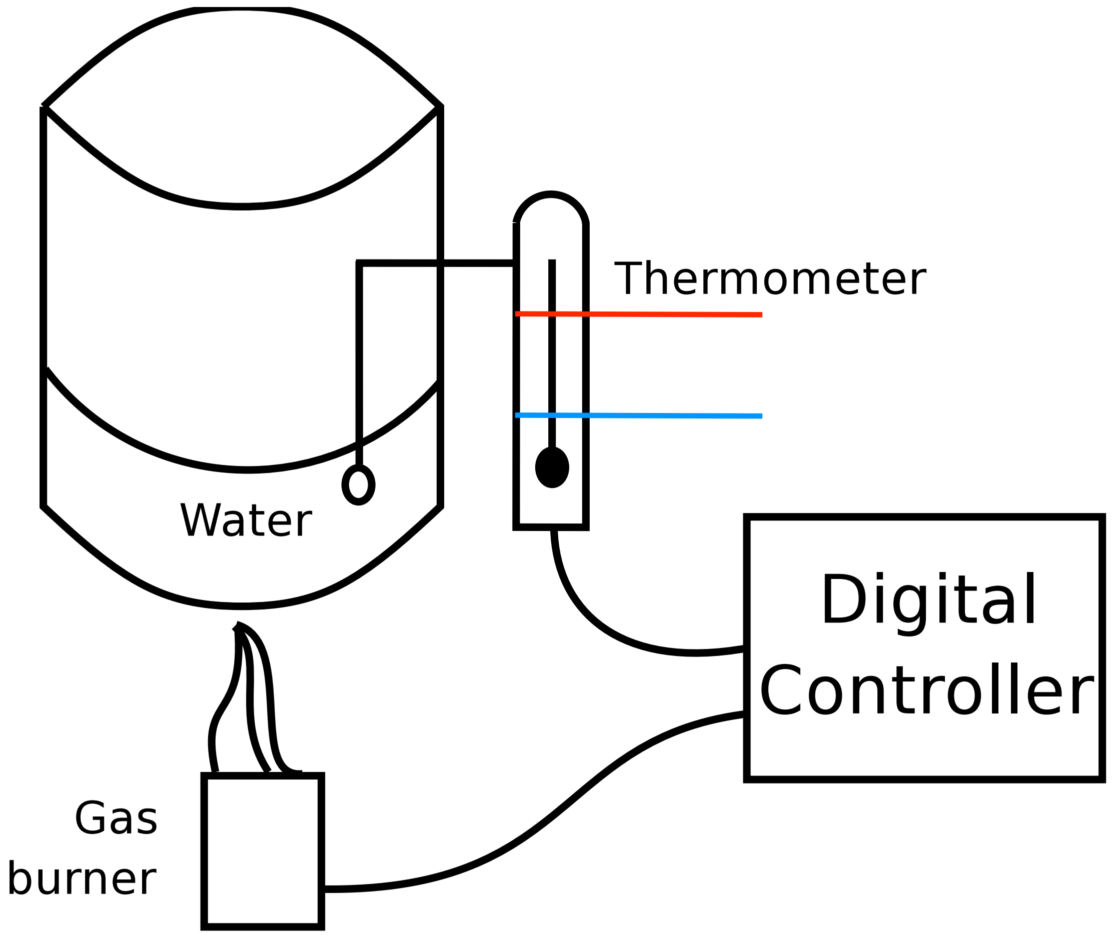
Content

- Controller synthesis : context - motivations
- Two-player game structures - Safety games (of perfect information)
- Imperfect information: motivations
- The lattice of antichains - CPre
- Applications : discrete time control of RHA, universality problem of NFA
- Conclusion & perspectives

Context - Motivations

Embedded systems



System

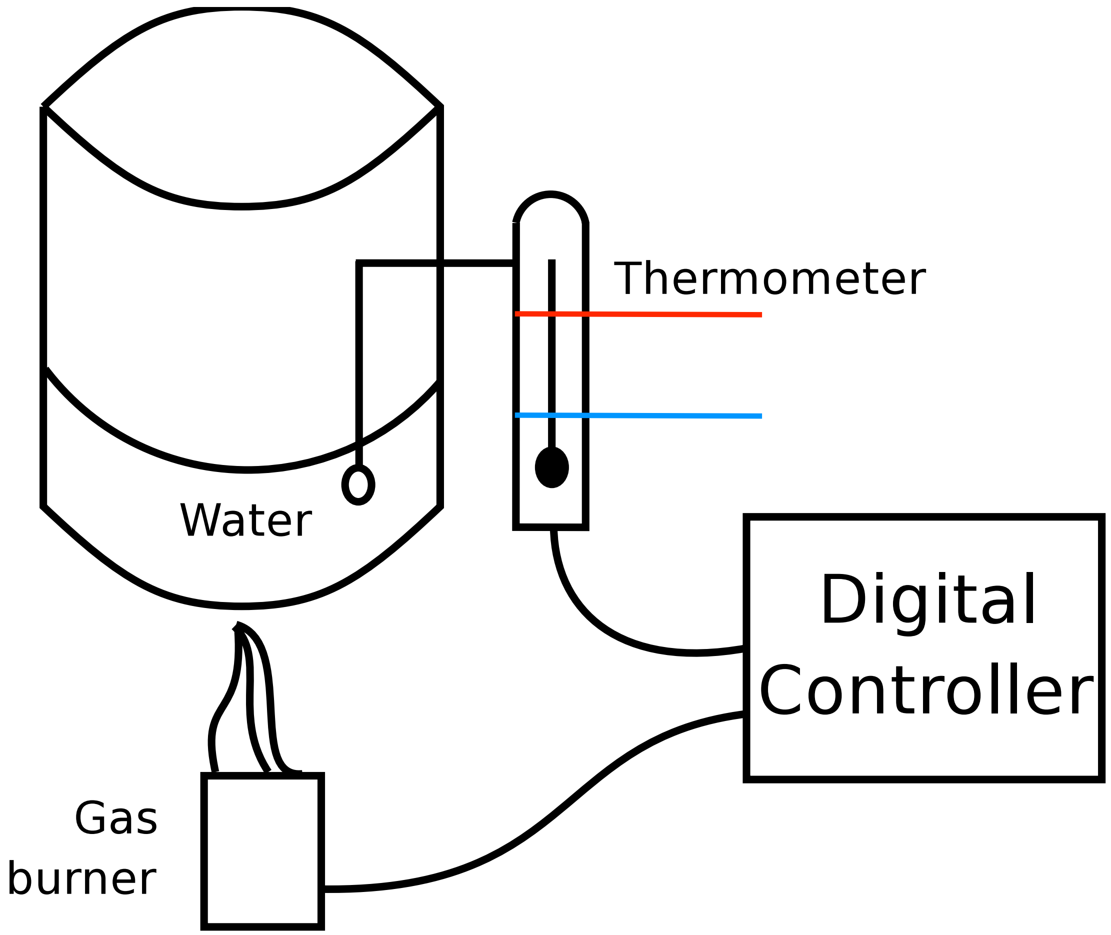
Hybrid systems mixe **discrete** and **continuous** components : non trivial interactions

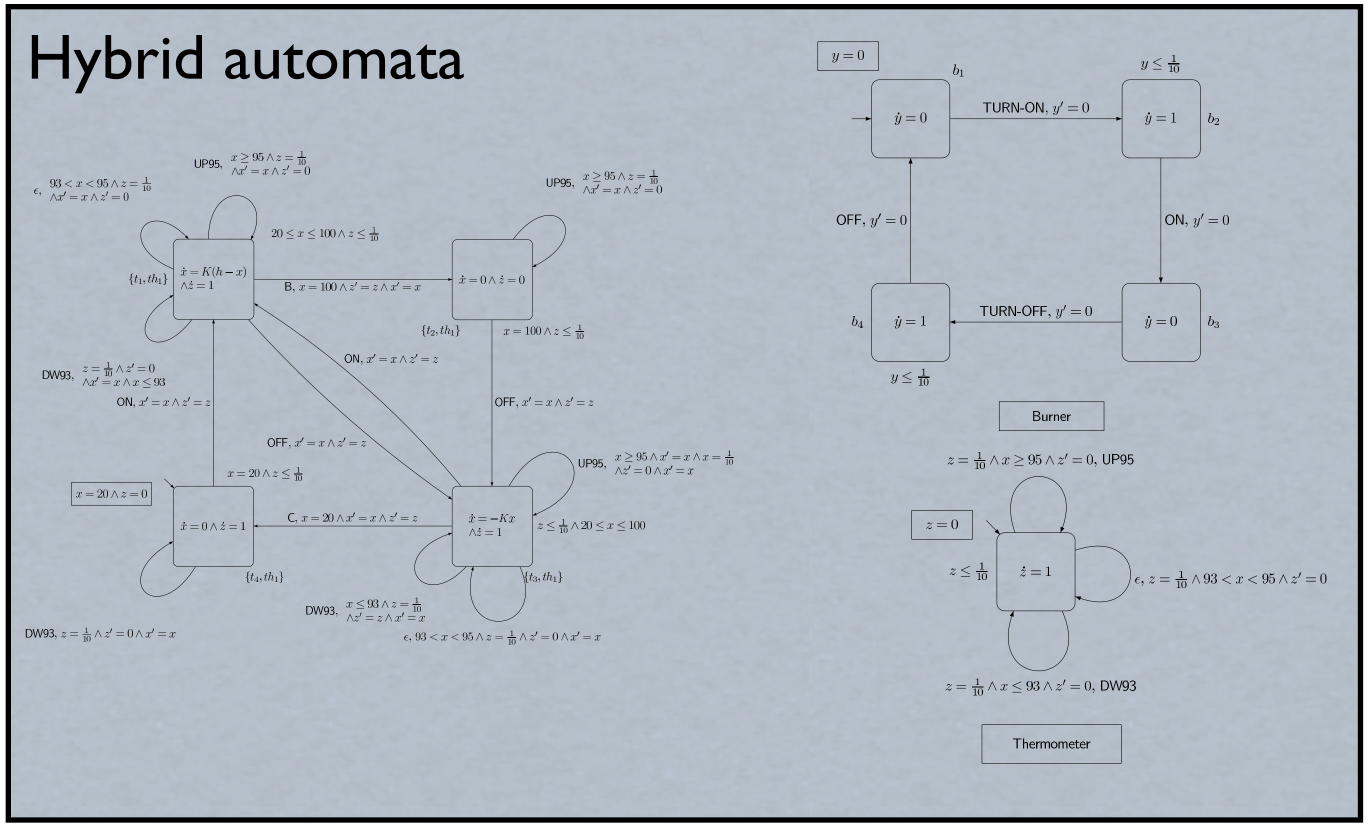
☛ **difficult to develp**

They are often **safety critical**

☛ **need for formal methods**

Controller Verification





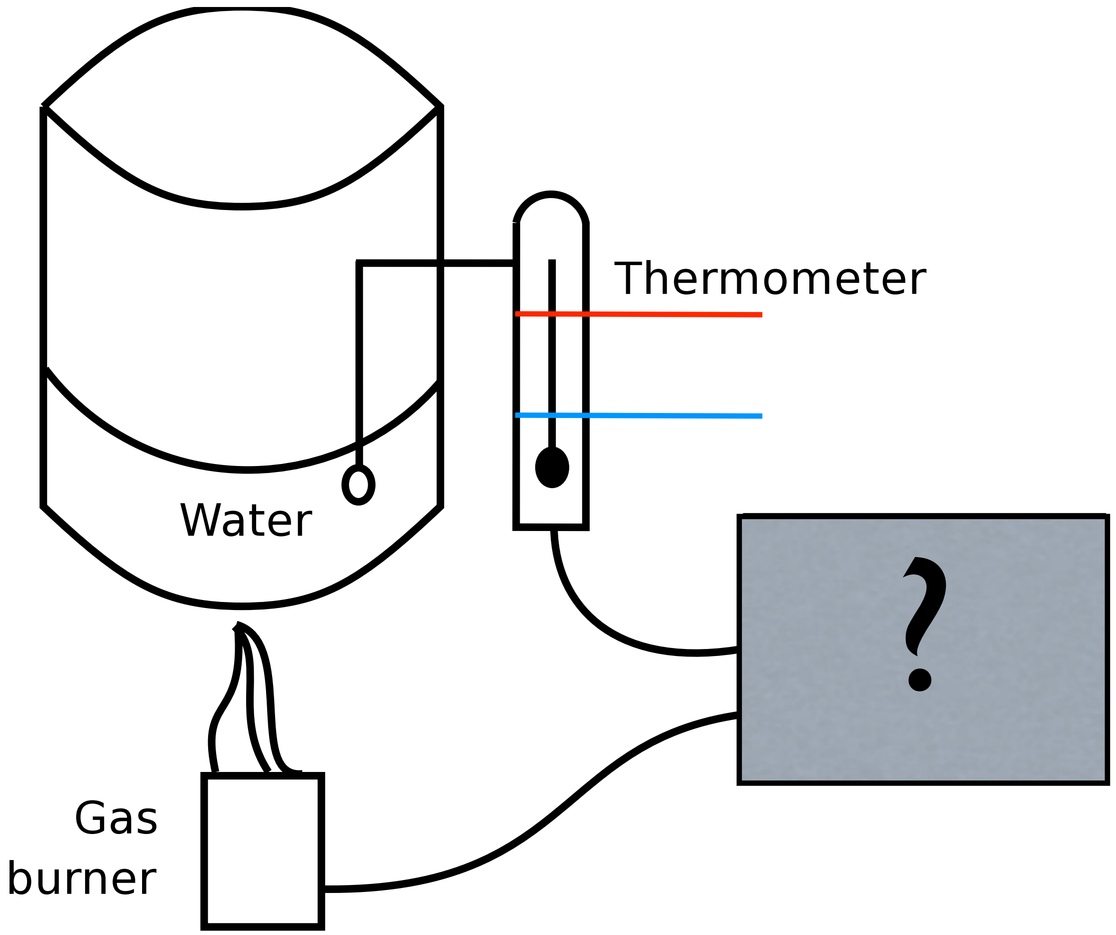
satisfies ?

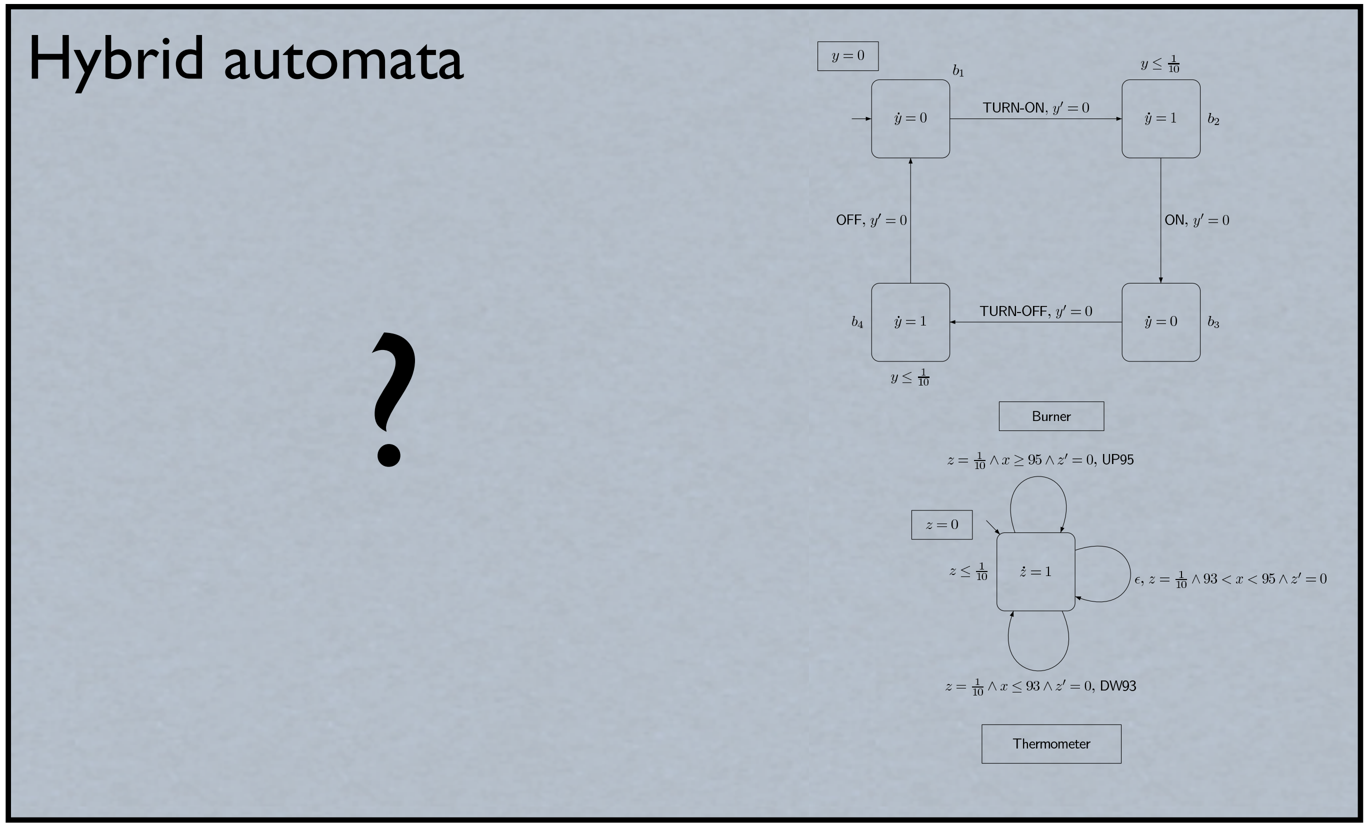
$\Box(\text{low} \leq x \leq \text{high})$

System

Math. model

Controller Synthesis





construct ? such that

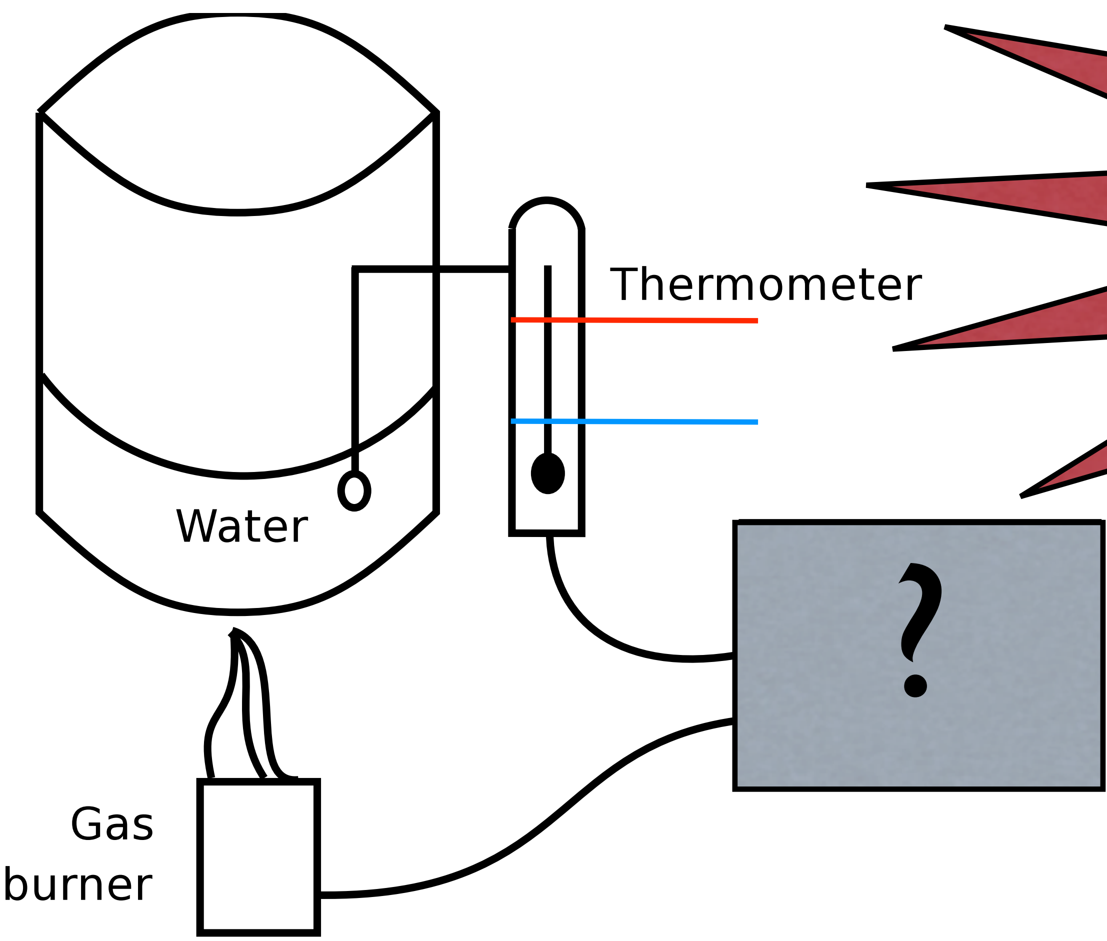
$$\Box(\text{low} \leq x \leq \text{high})$$

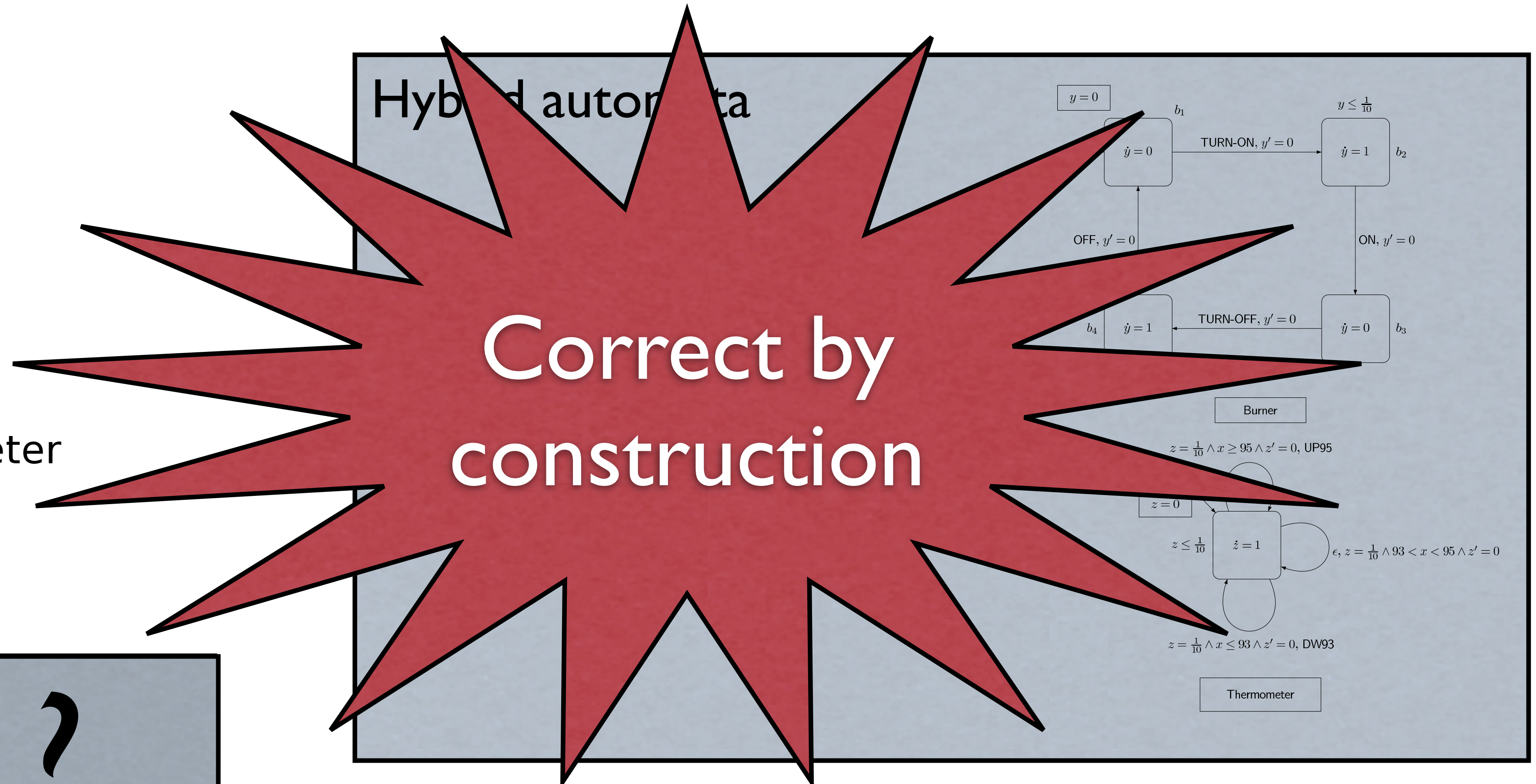
is satisfied

System

Math. model

Controller Synthesis





construct ? such that

$\Box(\text{low} \leq x \leq \text{high})$

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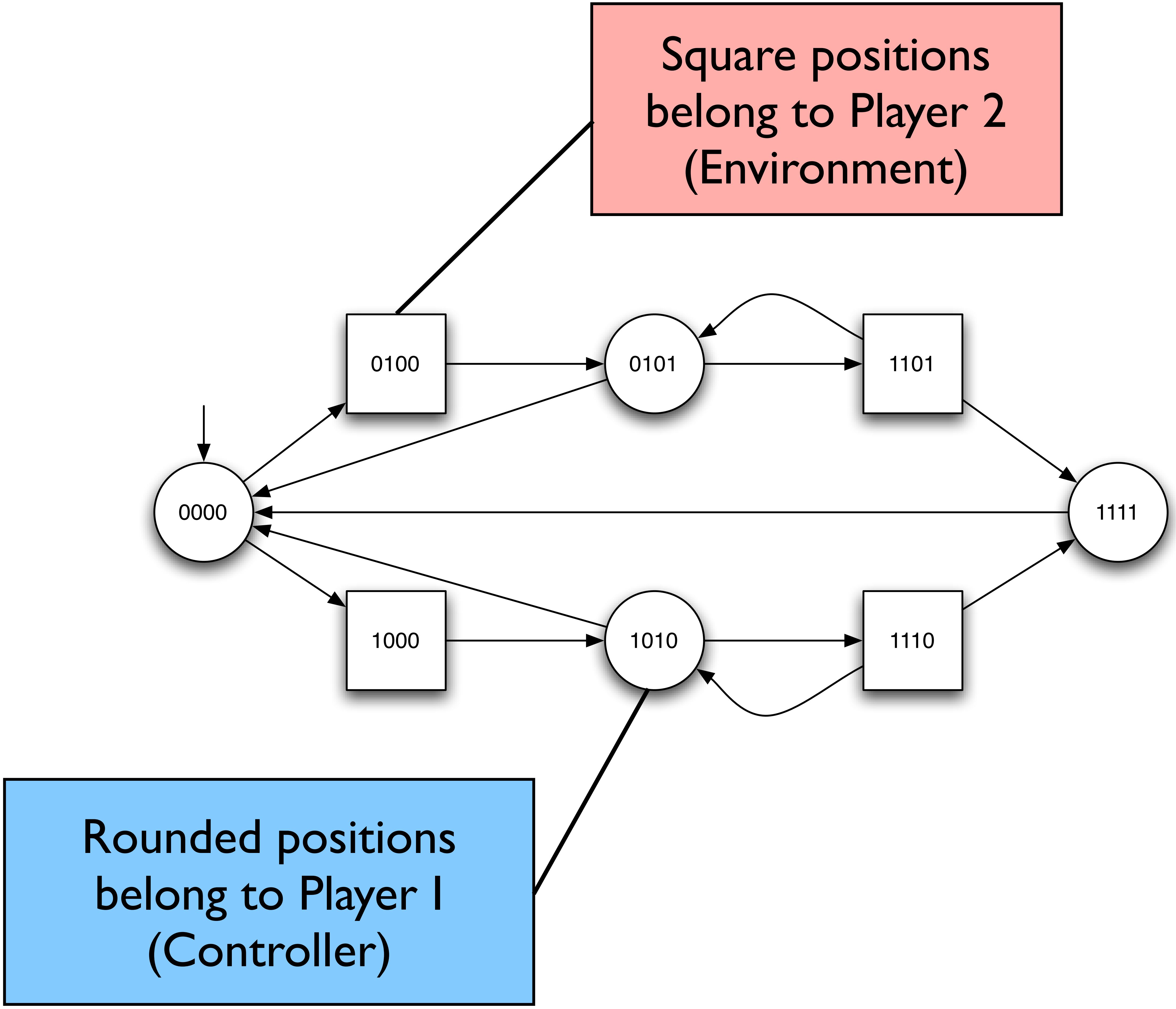
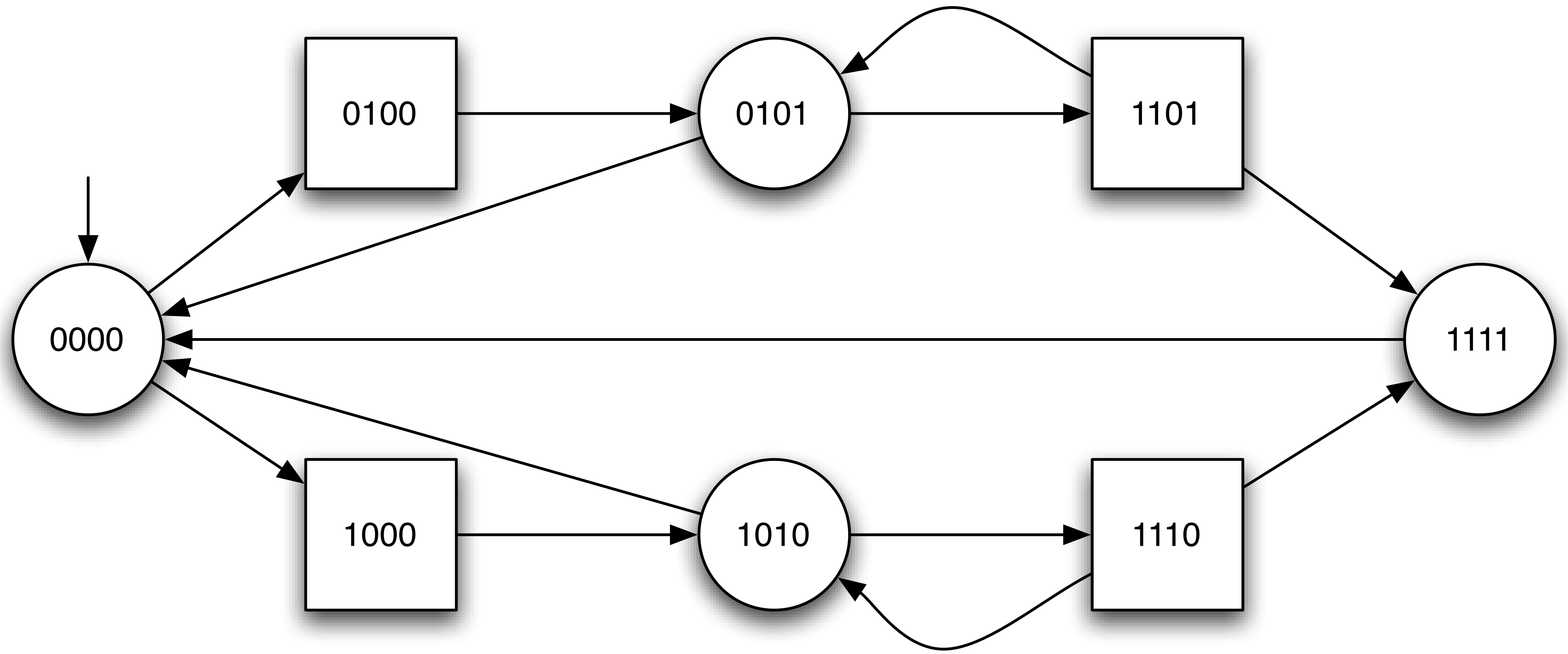
System

Math. model

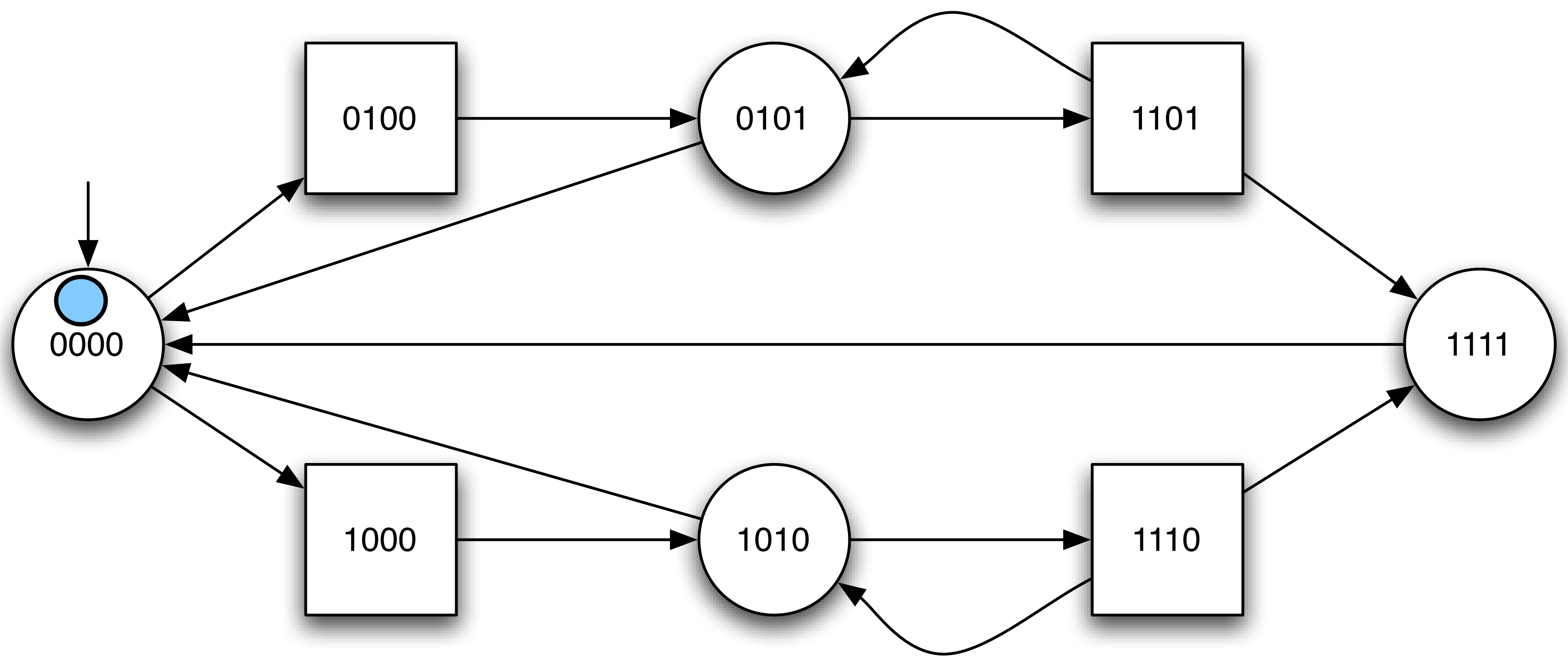
Two-player game structures


Square positions belong to Player 2 (Environment)
0100
0101
1101
0000
1111
1000
1010
1110
Rounded positions belong to Player I (Controller)

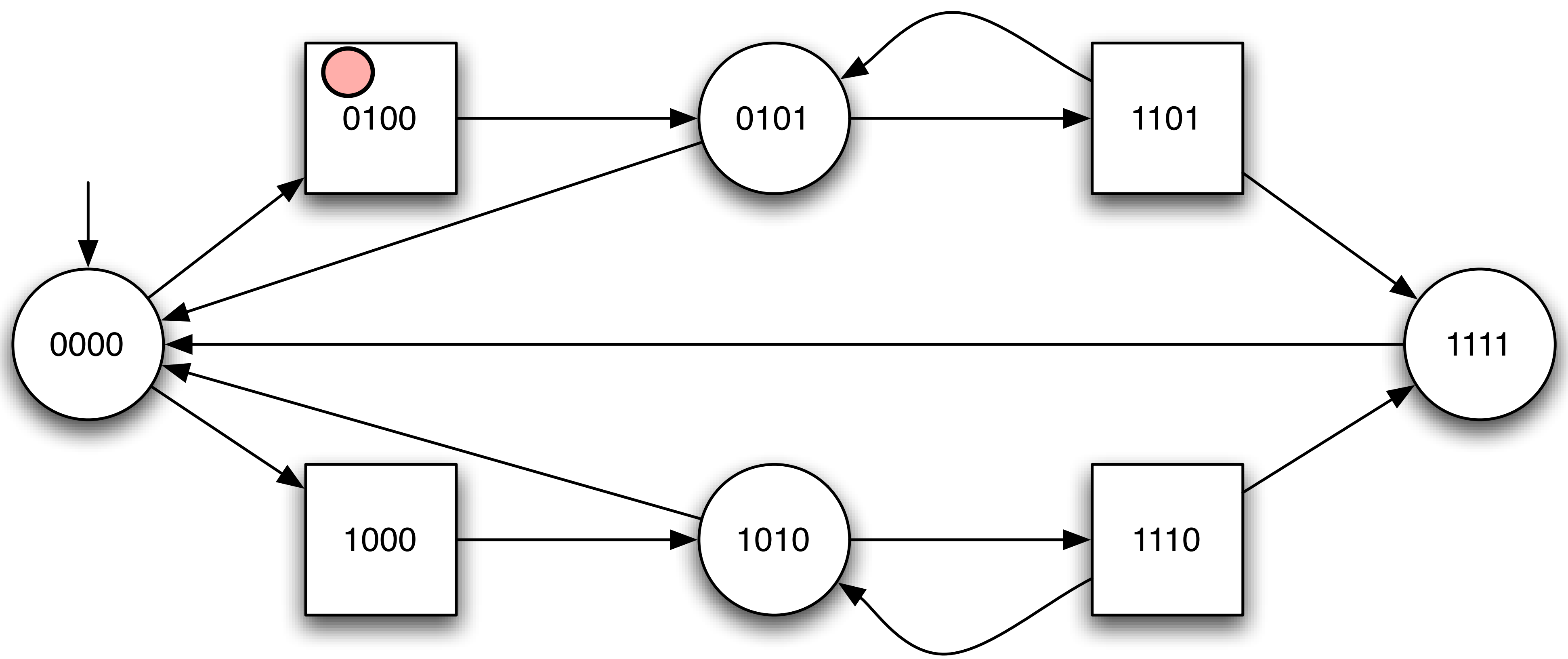
Rounded positions belong to Player 1
Square positions belong to Player 2



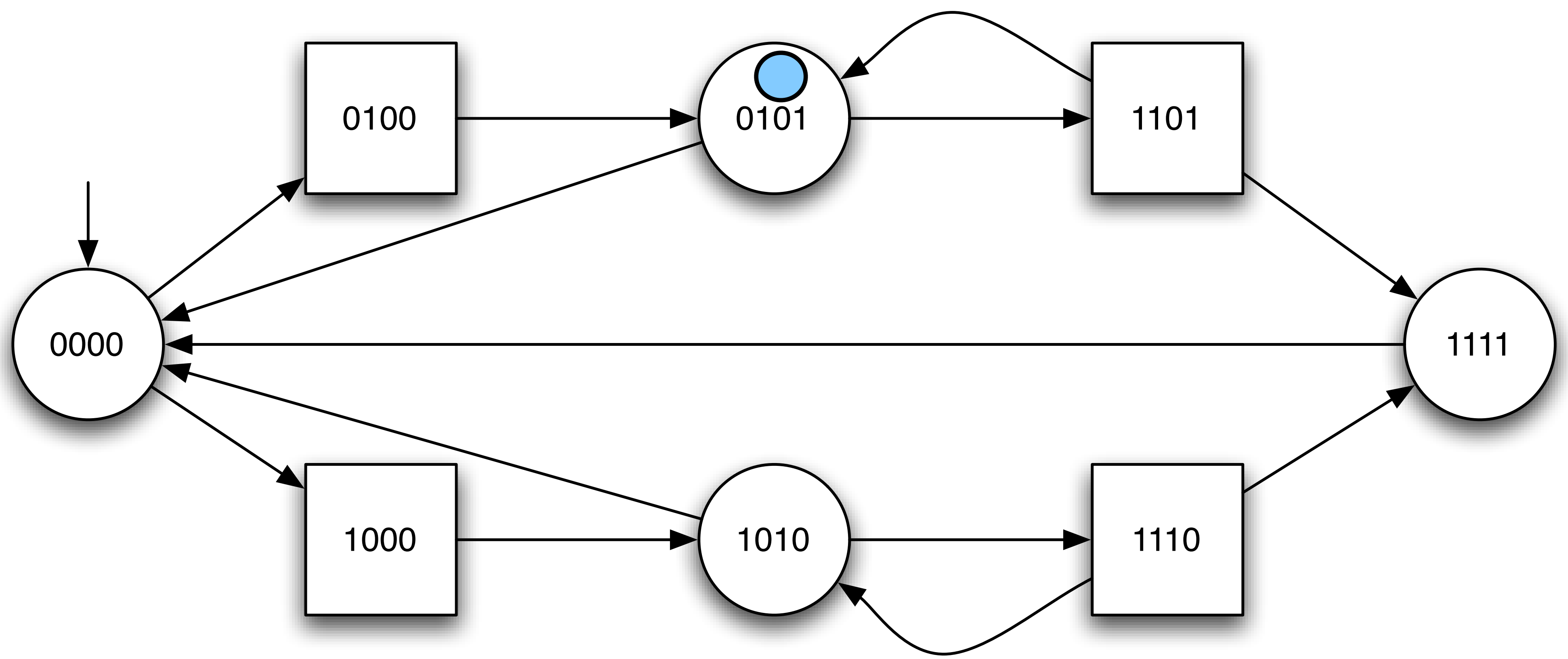
A game is played as follows: in each **round**, the game is in a **position**, if the game is in a rounded position, Player 1 resolves the **choice** for the next state, if the game is in a square position, Player 2 resolves the choice. The game is played for an **infinite number of rounds**.



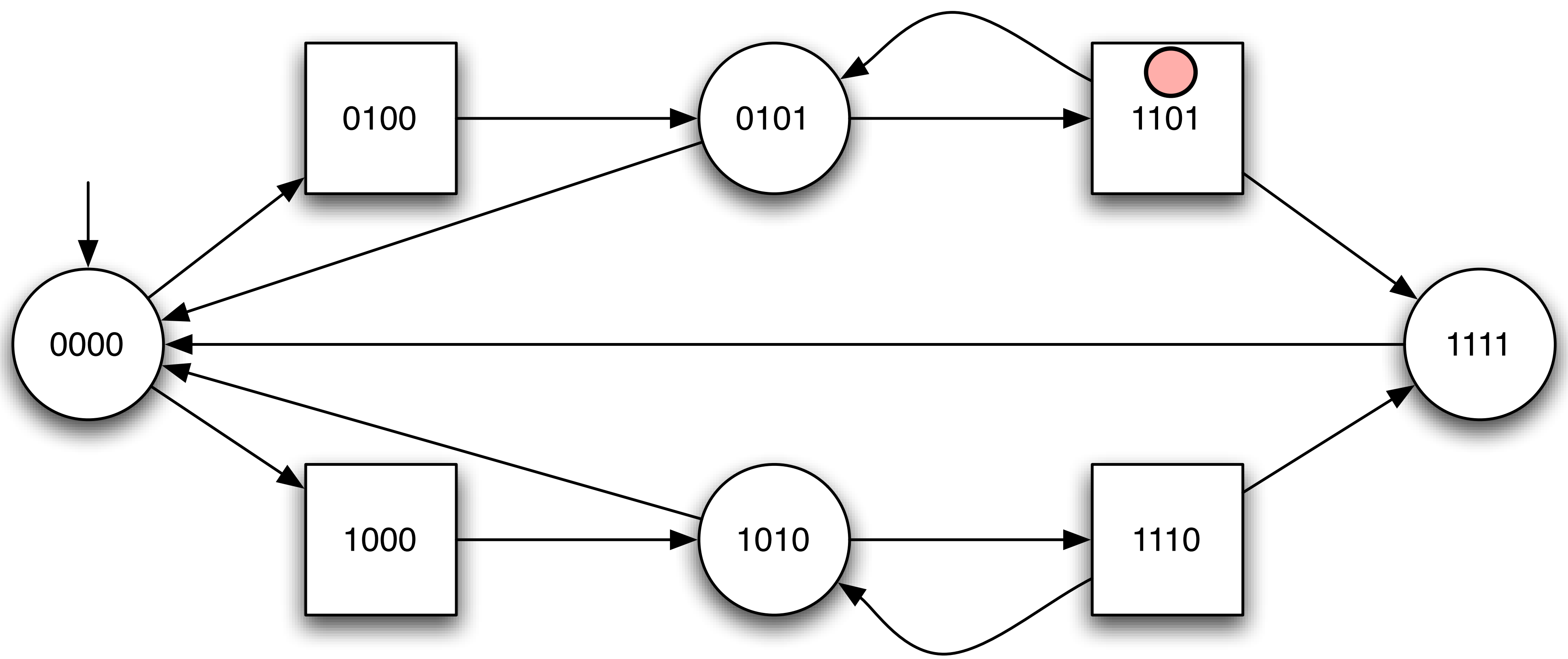
Play : 0000



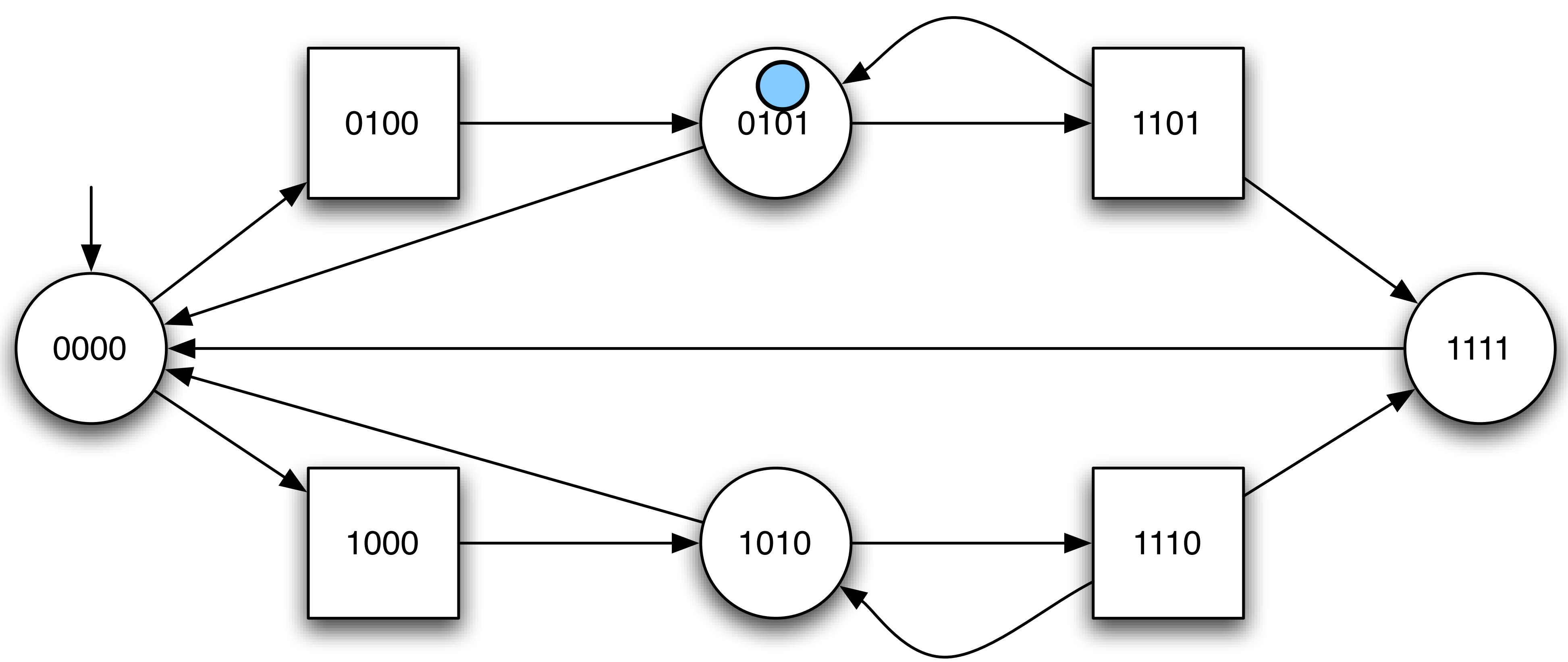
Play : 0000 0100



Play : 0000 0100 0101

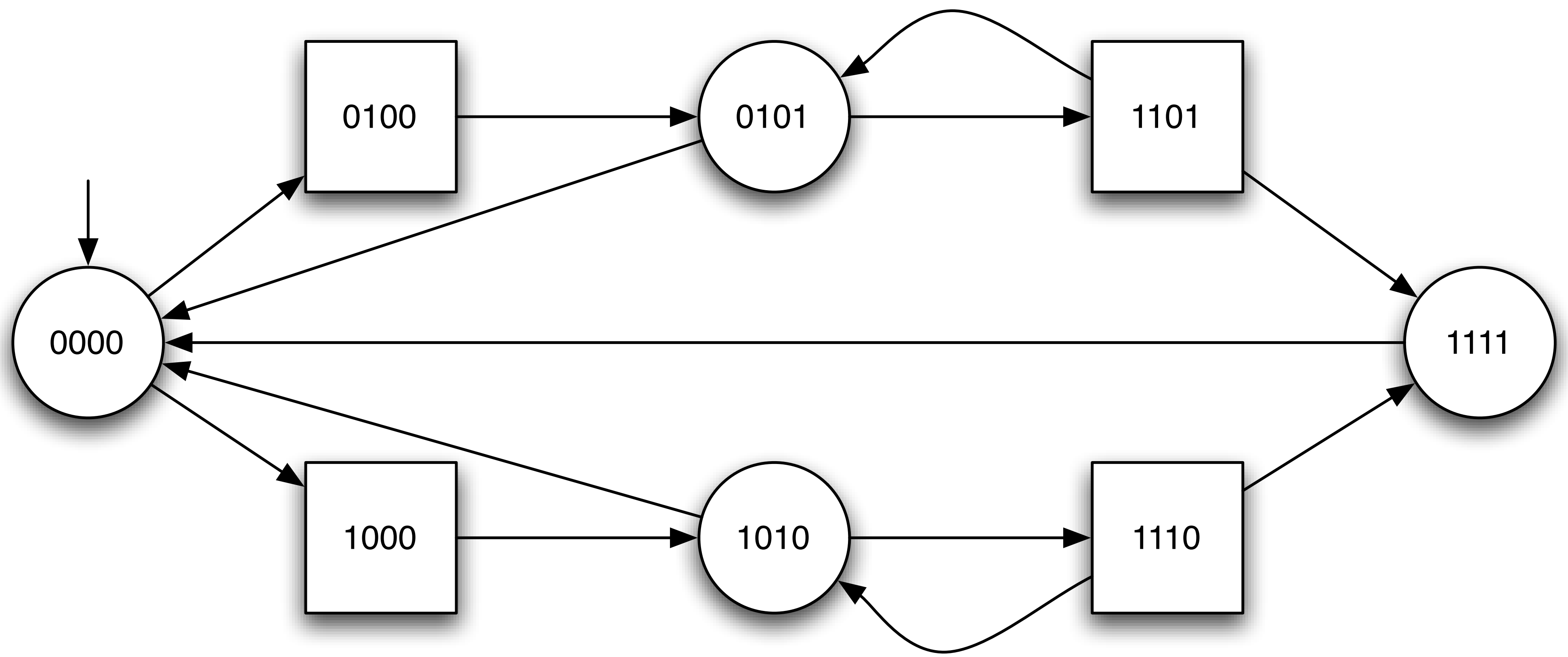


Play : 0000 0100 0101 1101



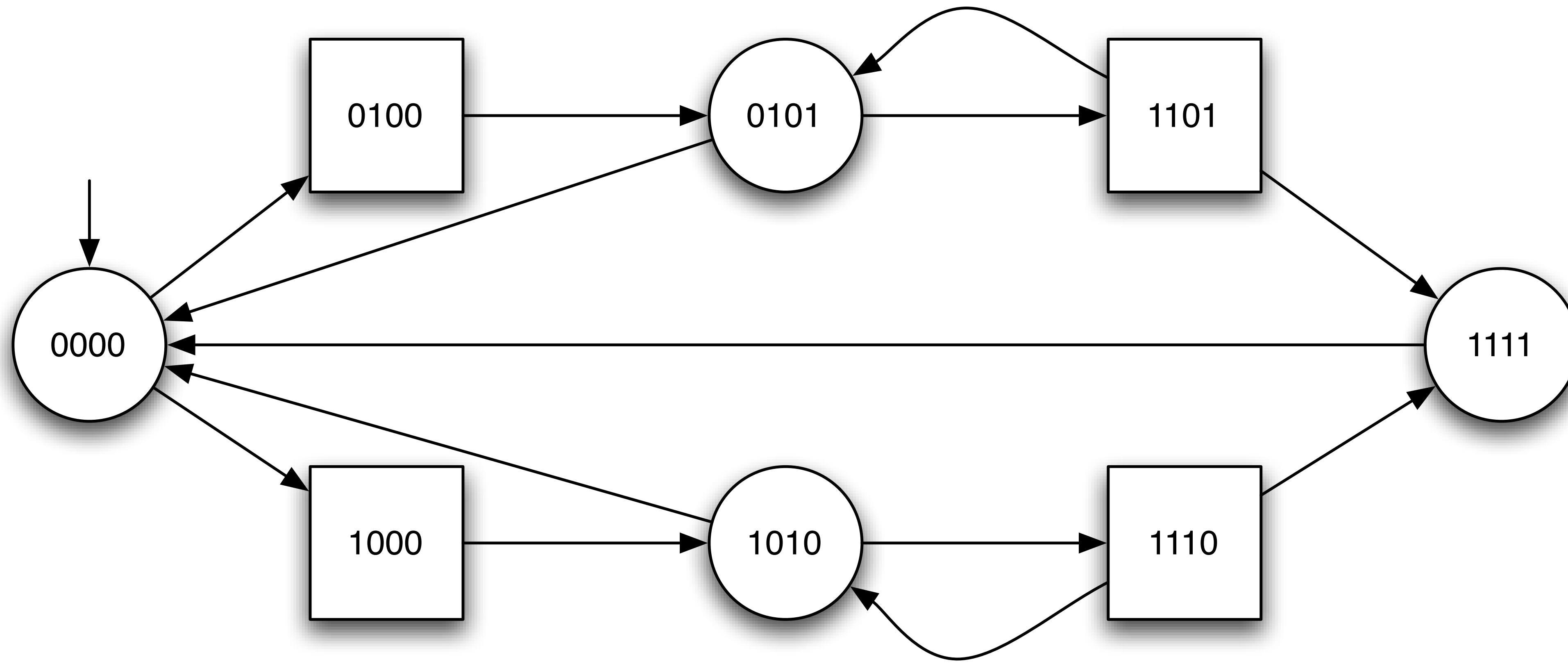
Play : 0000 0100 0101 1101 ...

Who is winning ?



Play : 0000 0100 0101 1101 ...

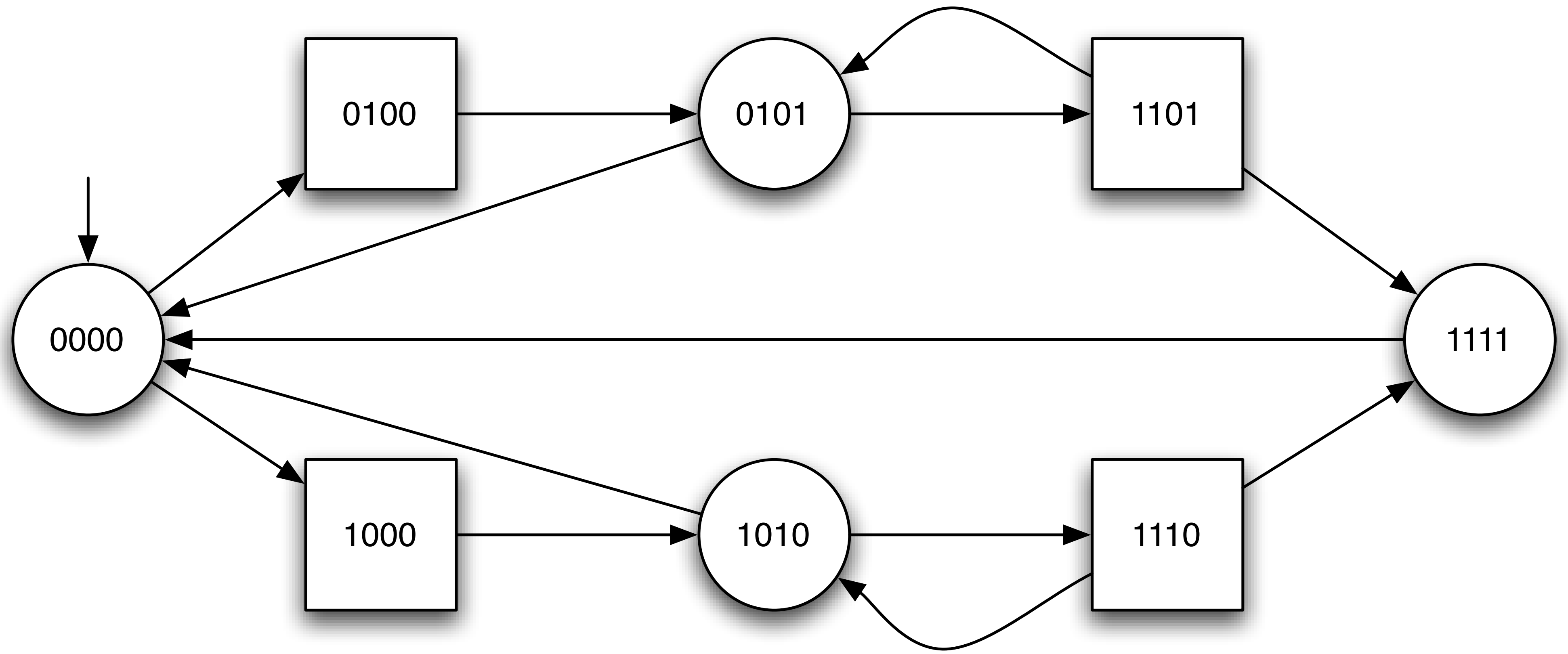
Who is winning ?



Play : 0000 0100 0101 1101 ...

Is this a **good** or a **bad** play for **Player *k*** ?

Who is winning ?



A **winning condition** (for Player *k*)
is a set of plays
$$W \subseteq (Q_1 \cup Q_2)^\omega$$

Game

=

Two-player game structure

+

Winning condition for Player *k*

Strategies

Players are playing **according to strategies.**

A **strategy for Player I** is a function that, given a *sequence of positions* (visited so far) that ends in a *Player I's position*, returns the *choice for the next position*.

$$\lambda_{\mathrm{I}}(\underbrace{0011\ 1001\ 1101\ \overbrace{0011}^{\text{Player I's position}}}_{\text{prefix of play}}) = \underbrace{1110}_{\text{Choice for the next position}}$$

Strategies

Players are playing **according to strategies.**

A **strategy for Player I** is a function that, given a *sequence of positions* (visited so far) that ends in a *Player I's position*, returns the *choice for the next position*.

Strategies for Player II
are defined symetrically

Outcome of strategies

If we **fix** a strategy for the two players and we let the two players apply their strategies, we get a play:

Outcome(λ1,λ2)=1100 0011 0001 0011 ...

If we fix a strategy **only** for Player I, we get a set of plays

Outcome(λ1)= $\bigcup_{\lambda_2}$ Outcome (λ1,λ2)

A strategy for Player I is **winning** for objective *W* iff

Outcome(λ1) ⊆ *W*

Outcome of strategies

A strategy for Player I is **winning** for objective W iff

$$\mathrm{Outcome}(\lambda_{\mathrm{I}}) \subseteq W$$

That is, *no matter how Player II resolves his choices*, when player I **plays according to** λ_{I} the resulting play belongs to W.

Player I can **force** the play to be in W.

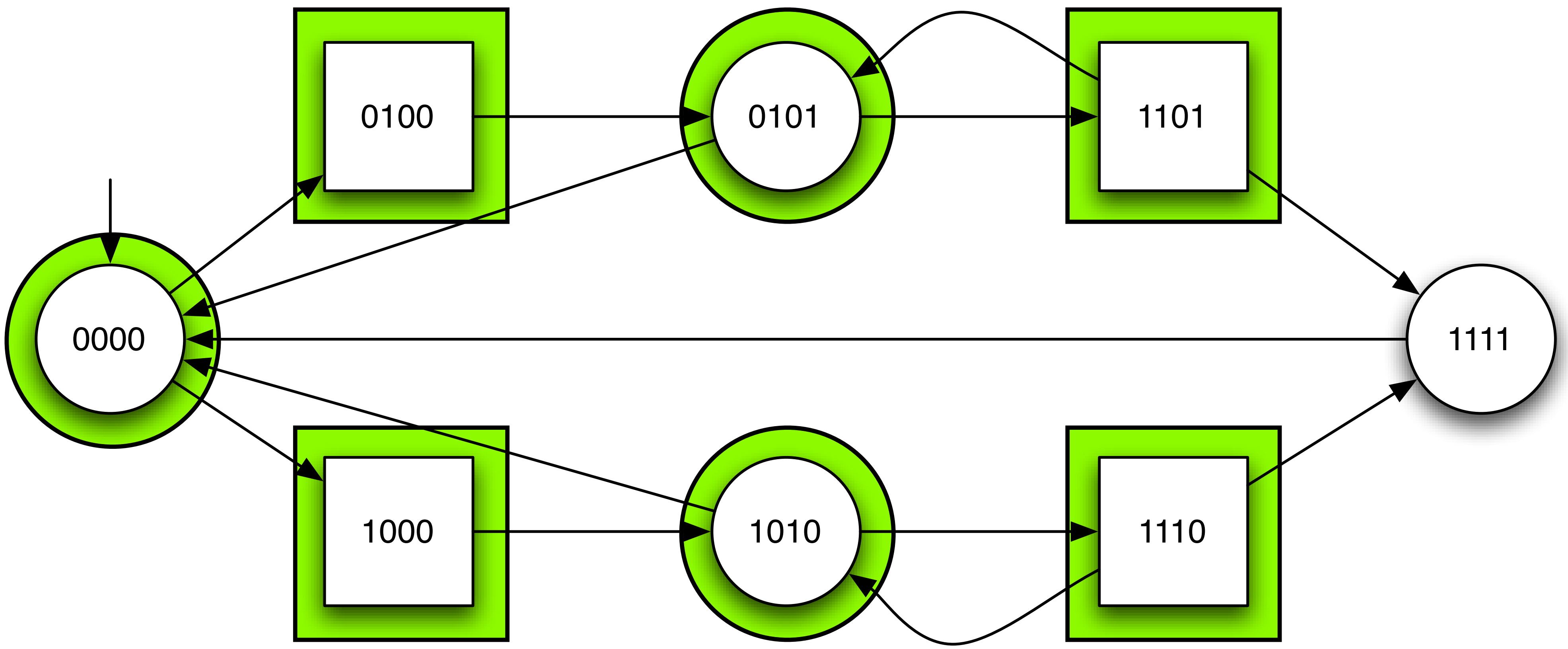
Winning strategies

=

Controllers that enforce winning plays

Safety Games

A Safety Game

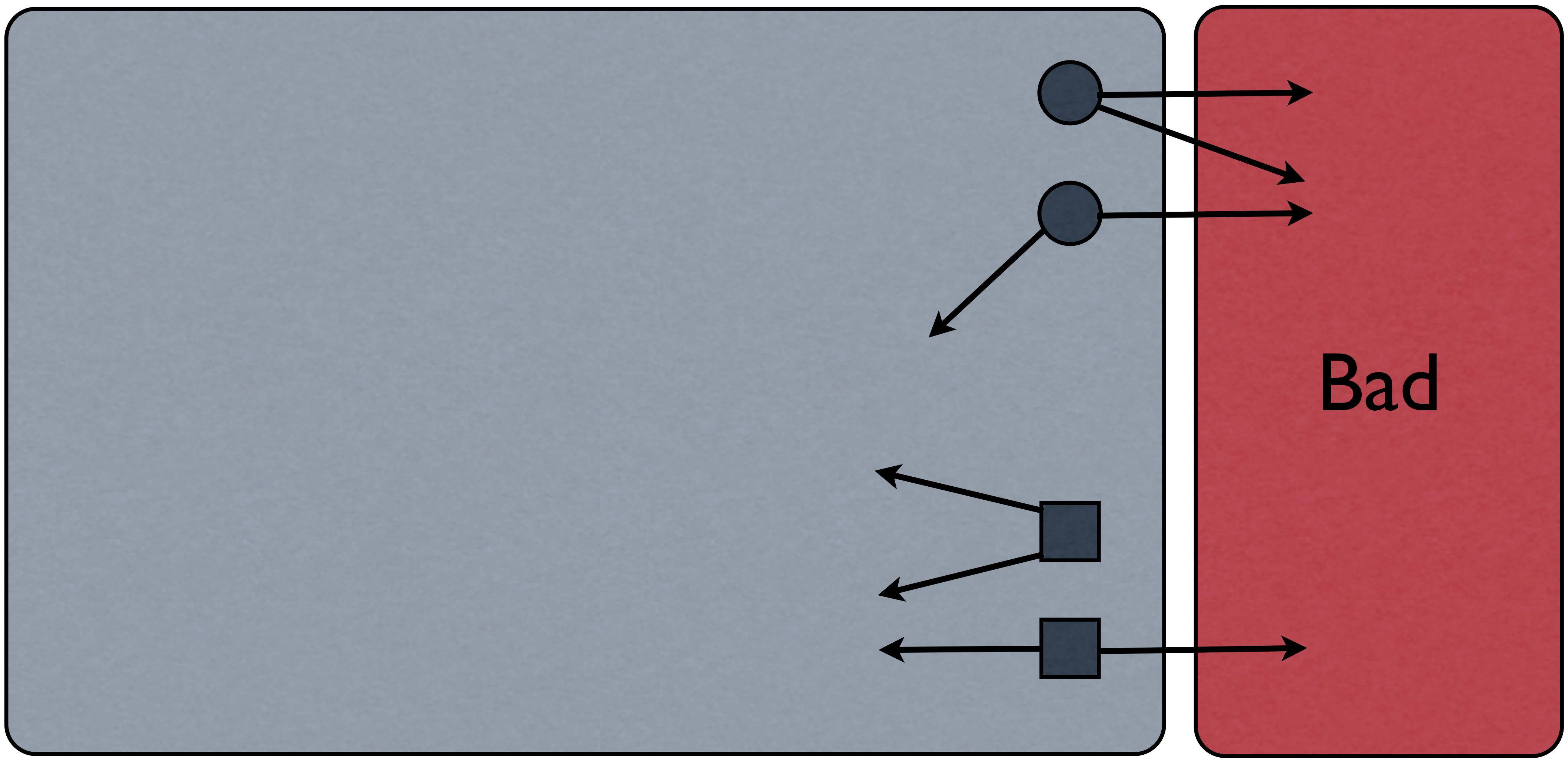


Does Player I, who owns the rounded positions, have a strategy (against any choices of Player II) to stay within the set of states

$\boxed{Q \setminus \{1111\}}$?

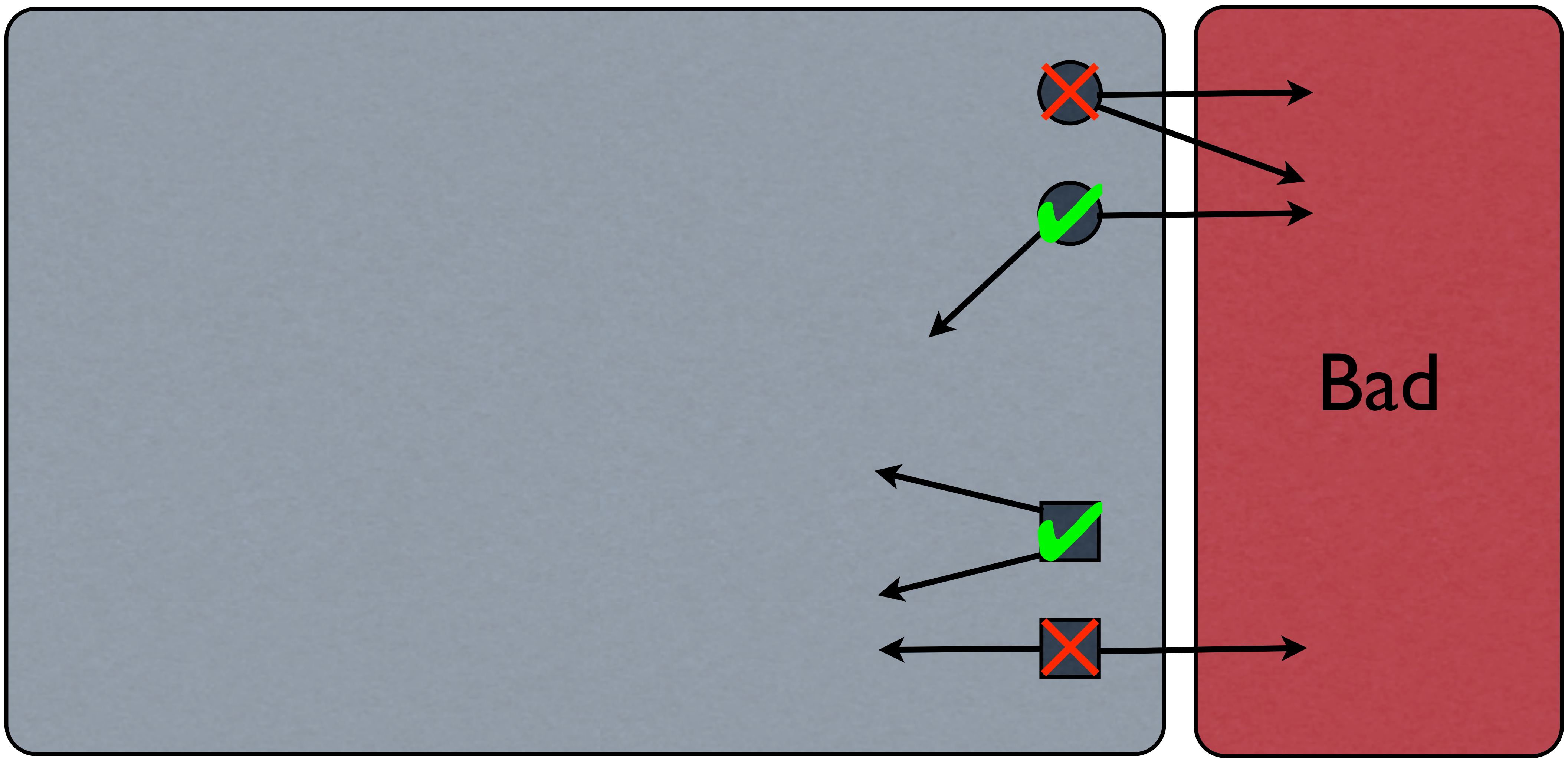
Symbolic algorithms to solve games

Symbolic algorithm for safety games



From where can Player I avoid Bad ?

Symbolic algorithm for safety games



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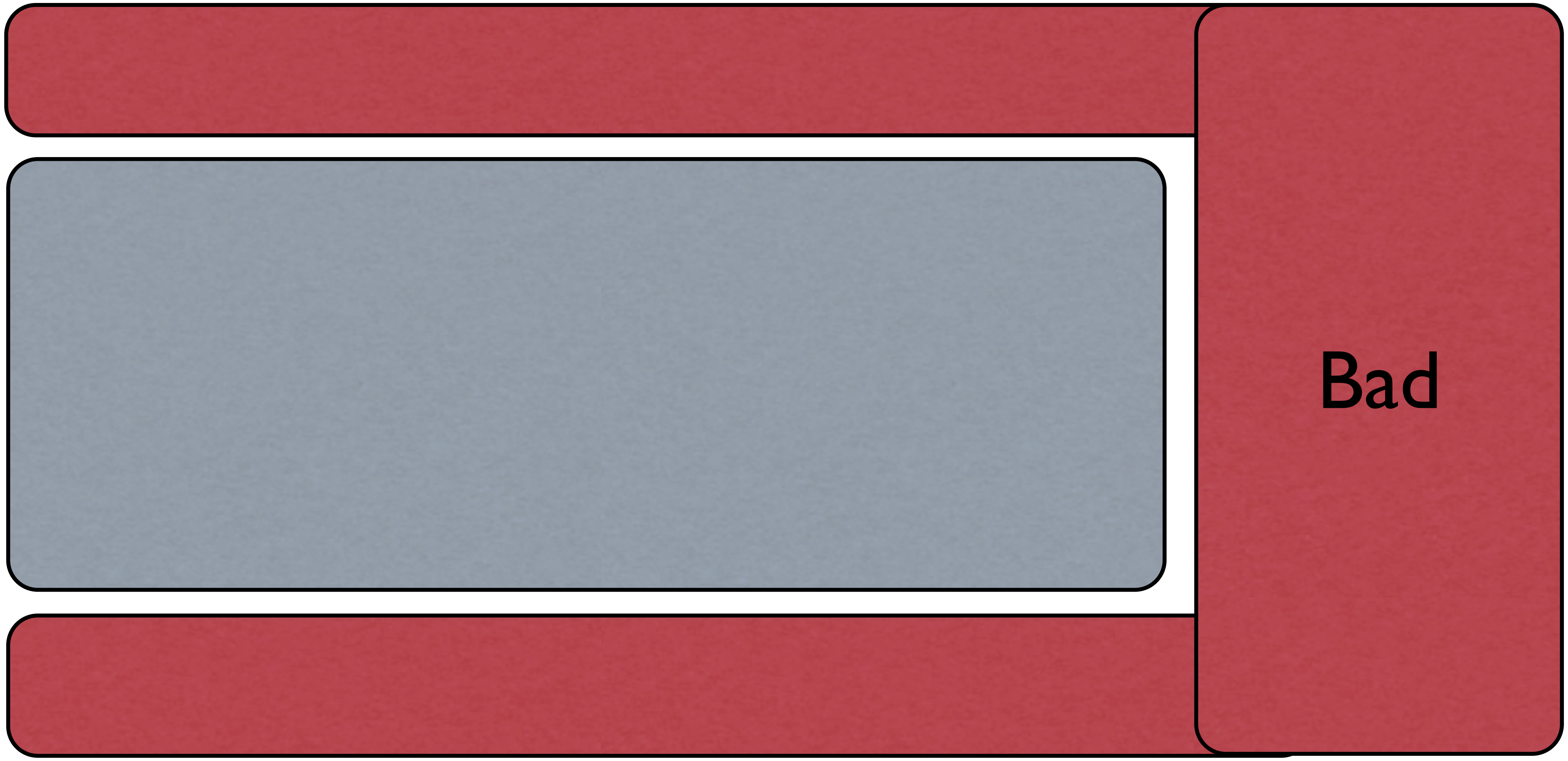

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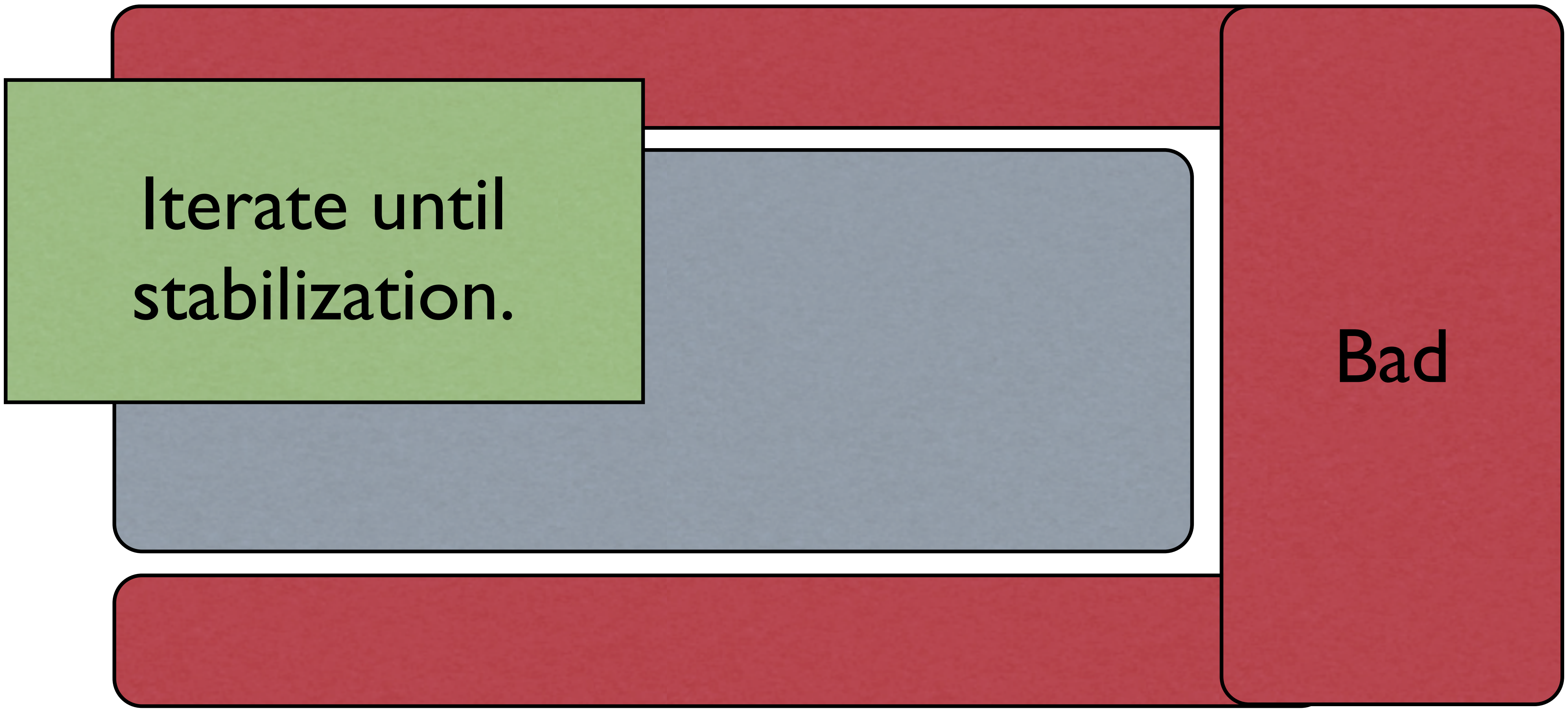
Symbolic algorithm for safety games



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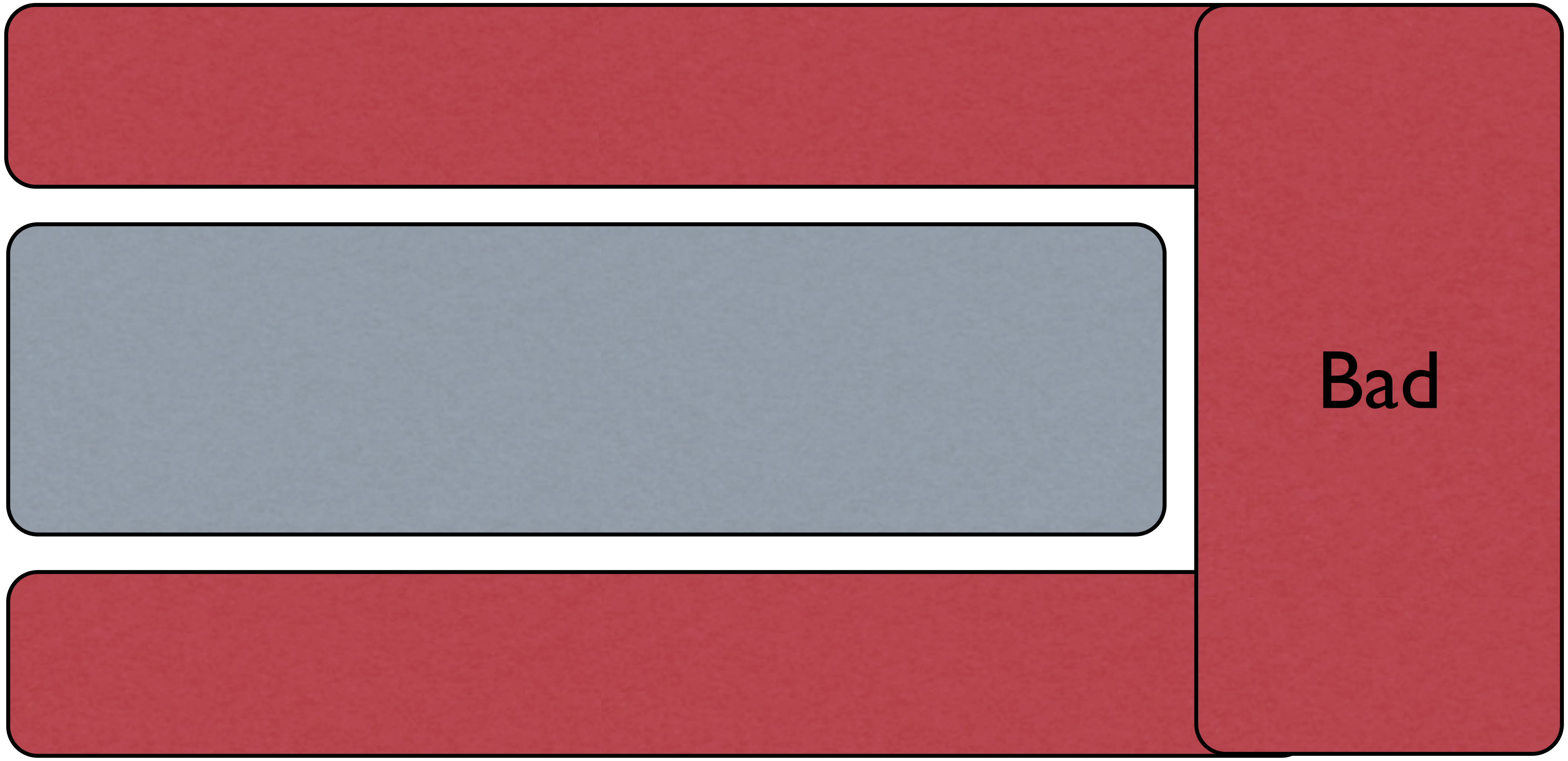
Symbolic algorithm for safety games



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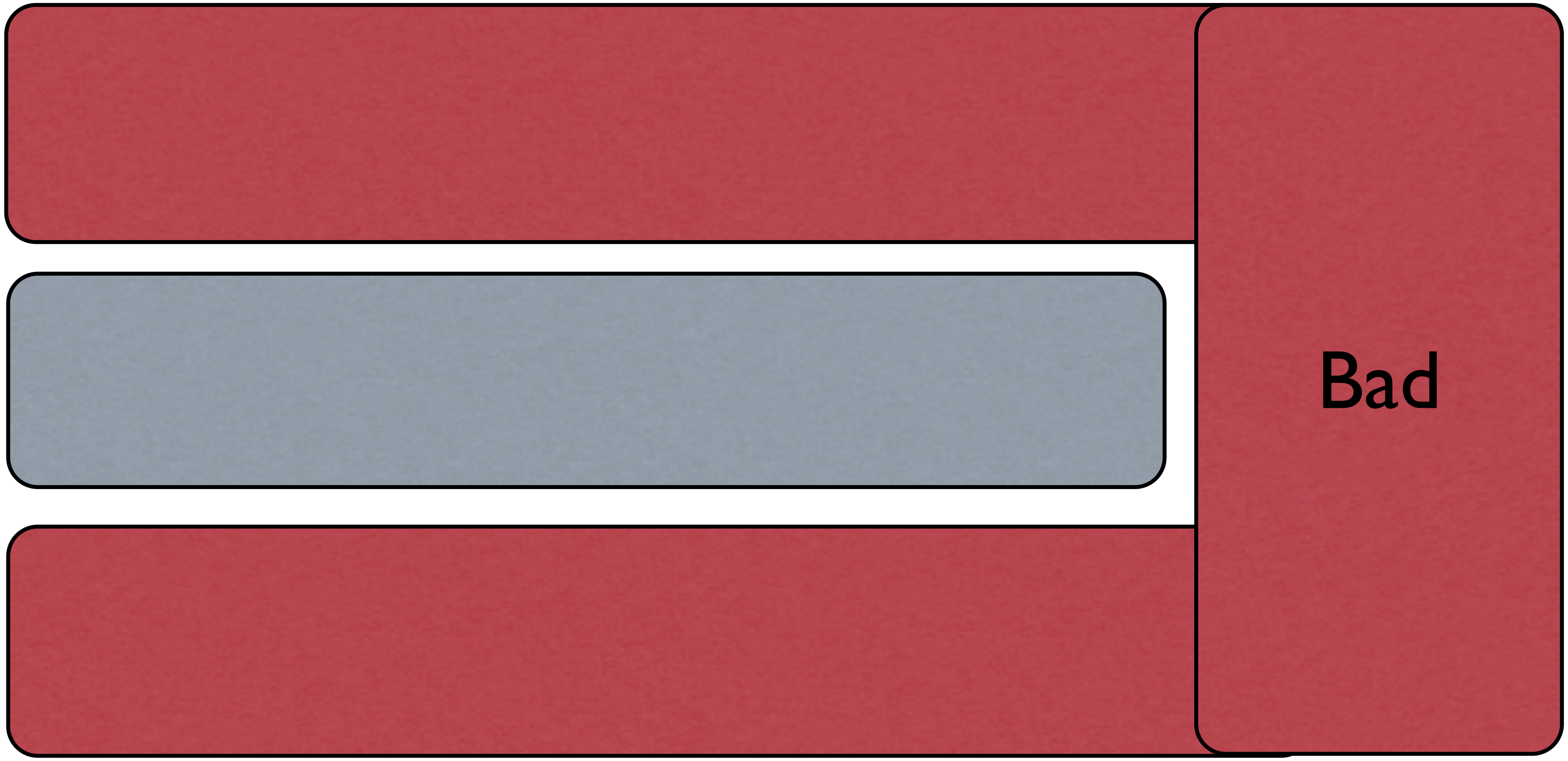
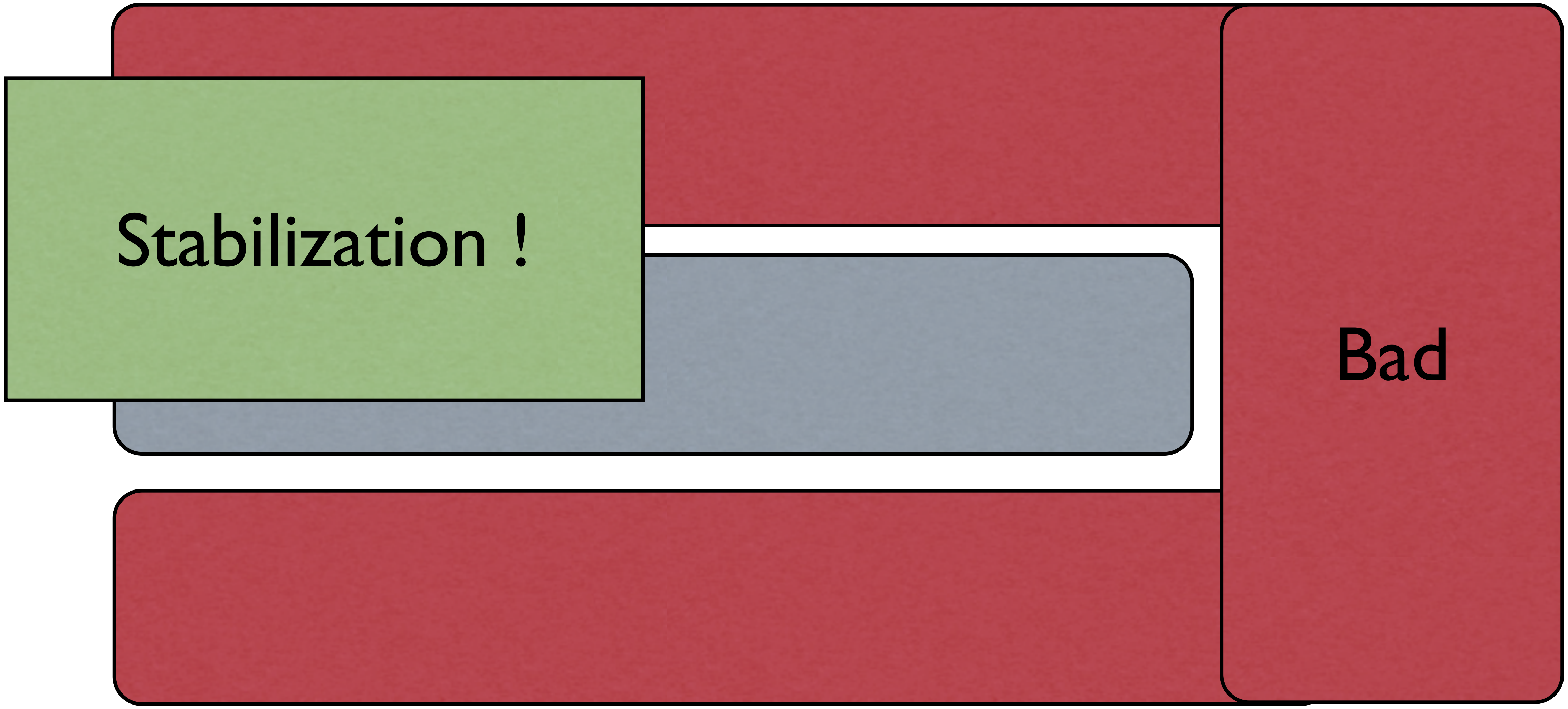
Symbolic algorithm for safety games



From where can Player I avoid Bad ?

Symbolic algorithm for safety games



From where can Player I avoid Bad ?

Symbolic algorithm for safety games

This is exactly the set of states where Player I has a strategy to avoid the bad states.

Player I Controllable Predecessors

X is a set of positions

$$1\mathsf{CPre}_G(X) = \{q \in Q_1 \mid \exists q' : \delta(q, q') \wedge q' \in X\} \cup \{q \in Q_2 \mid \forall q' : \delta(q, q') : q' \in X\}$$

Set of Player I positions where she has a choice of successor that lies in *X*

Set of Player II positions where all her choices for successors lie in *X*

Player 1 Controllable Predecessors

$$\mathsf{1CPre}_G(X) = \{q \in Q_1 \mid \exists q' : \delta(q, q') \wedge q' \in X\} \cup \{q \in Q_2 \mid \forall q' : \delta(q, q') : q' \in X\}$$

Symmetrically

$$\mathsf{2CPre}_G(X) = \{q \in Q_2 \mid \exists q' : \delta(q, q') \wedge q' \in X\} \cup \{q \in Q_1 \mid \forall q' : \delta(q, q') : q' \in X\}$$

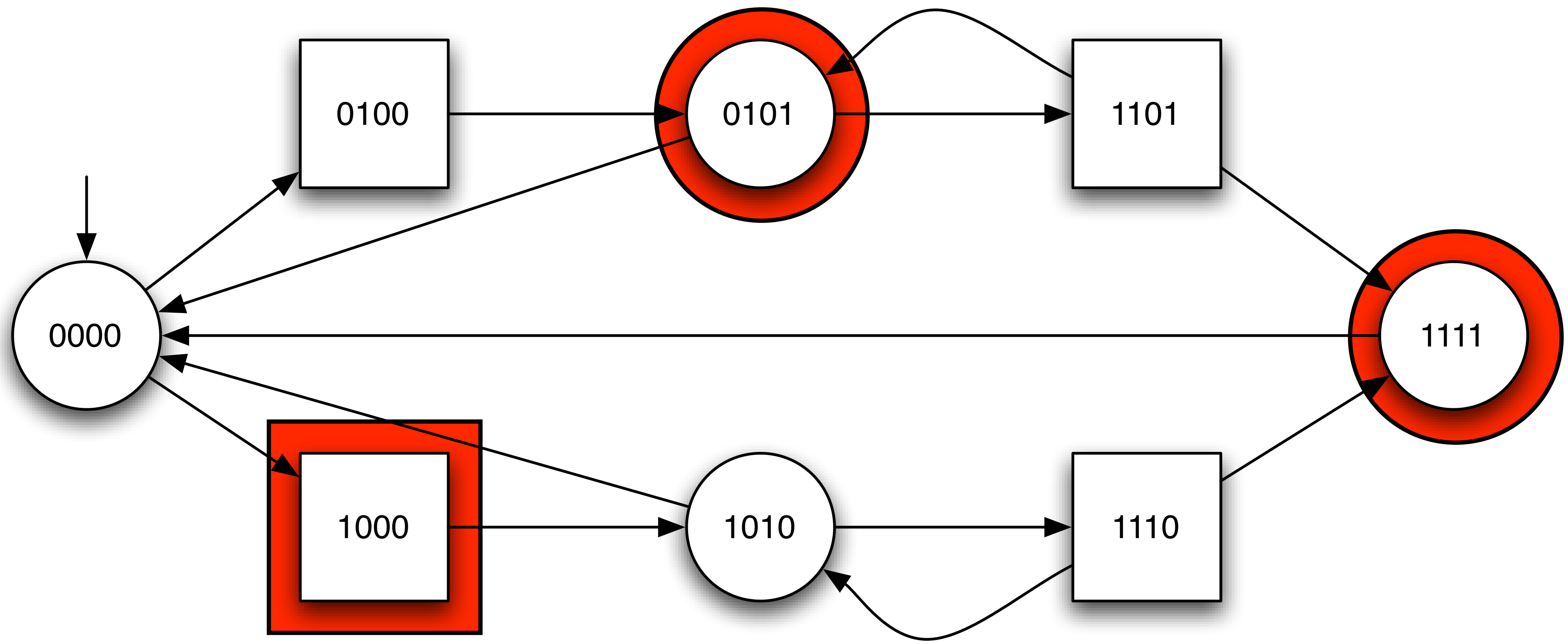
Player 1 Controllable Predecessors

$$1\mathsf{CPre}_G(X) = \{q \in Q_1 \mid \exists q' : \delta(q,q') \wedge q' \in X\} \cup \{q \in Q_2 \mid \forall q' : \delta(q,q') : q' \in X\}$$

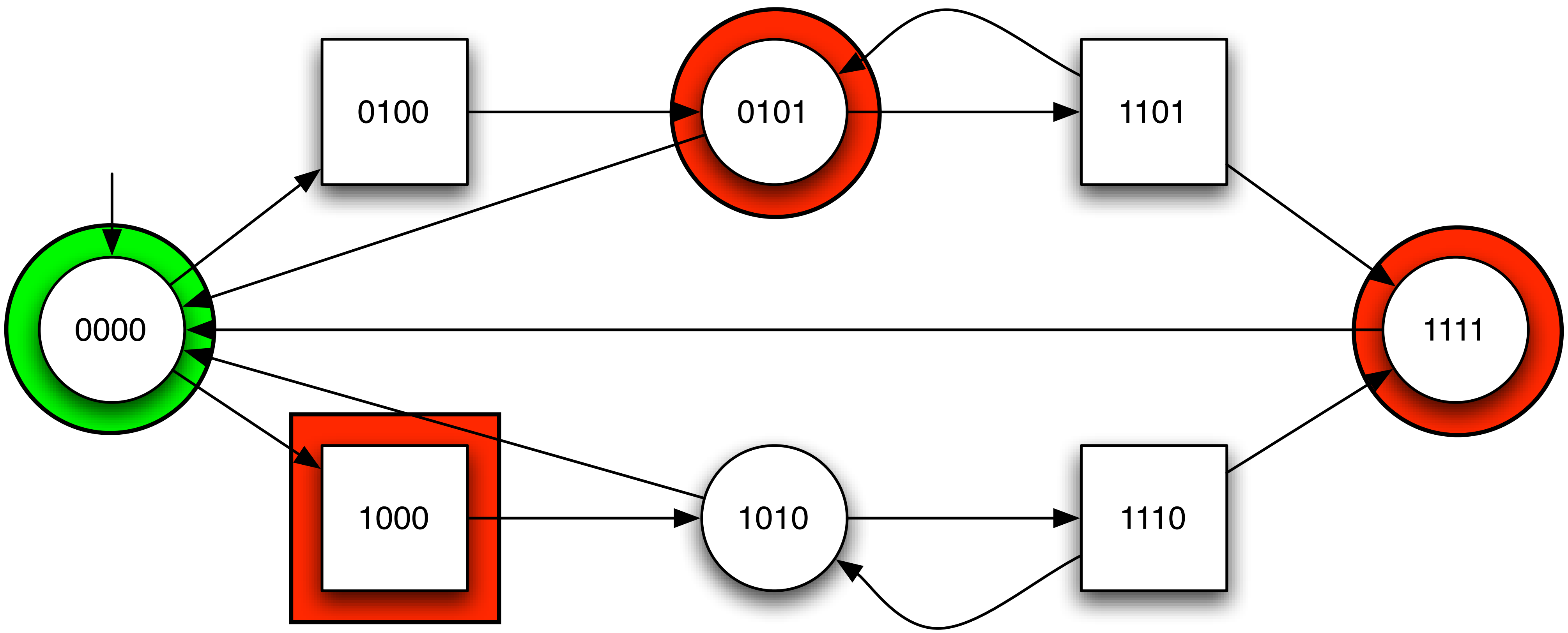
Monotonic functions over $\langle 2^{Q_1 \cup Q_2}, \subseteq \rangle$

$$2\mathsf{CPre}_G(X) = \{q \in Q_2 \mid \exists q' : \delta(q,q') \wedge q' \in X\} \cup \{q \in Q_1 \mid \forall q' : \delta(q,q') : q' \in X\}$$

Complete lattice



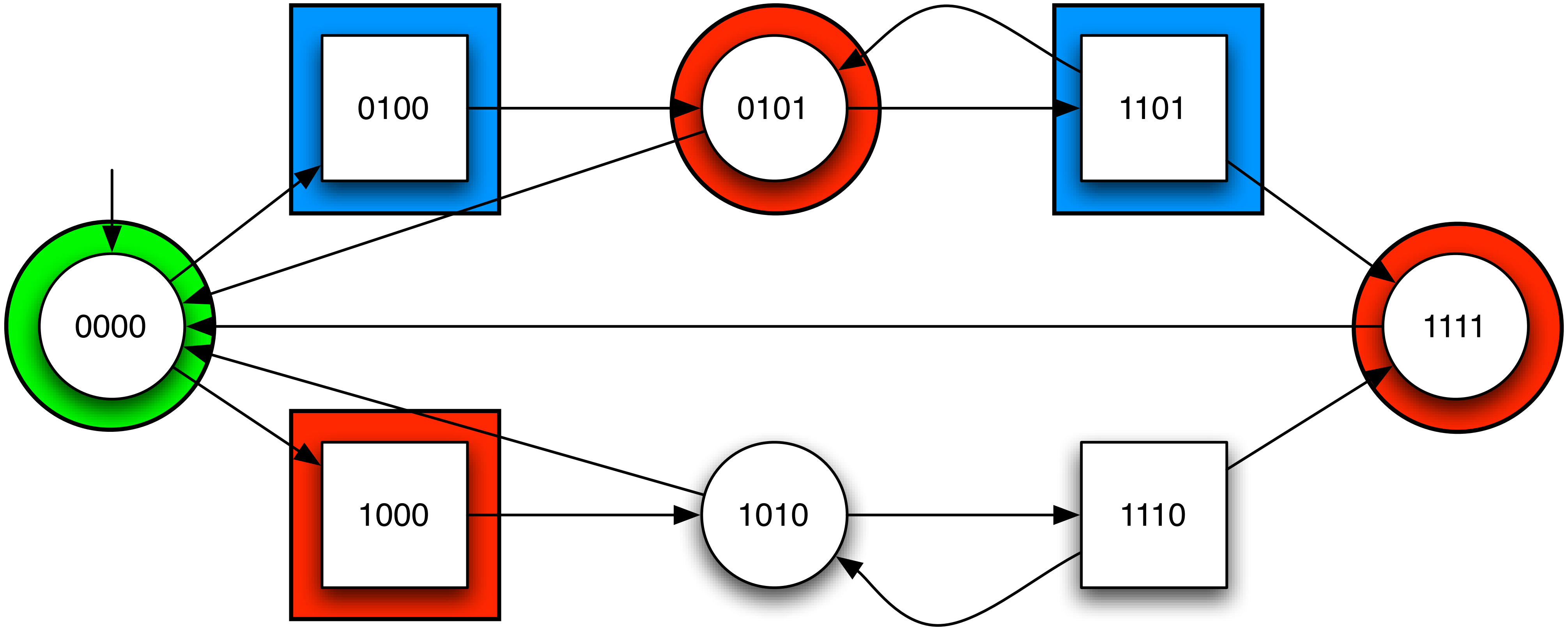
$$X = \boxed{\{1000, 0101, 1111\}}$$



$$X = \boxed{\{1000, 0101, 1111\}}$$

$$1\mathsf{CPre}(X) = \boxed{\{0000\}} \cup \{0100, 1101\}$$

Rounded positions,
there exists a red successor



$$X = \{1000, 0101, 1111\}$$

$$\mathsf{1CPre}(X) = \{0000\} \cup \{0100, 1101\}$$

Rounded positions,
there exists a red successor

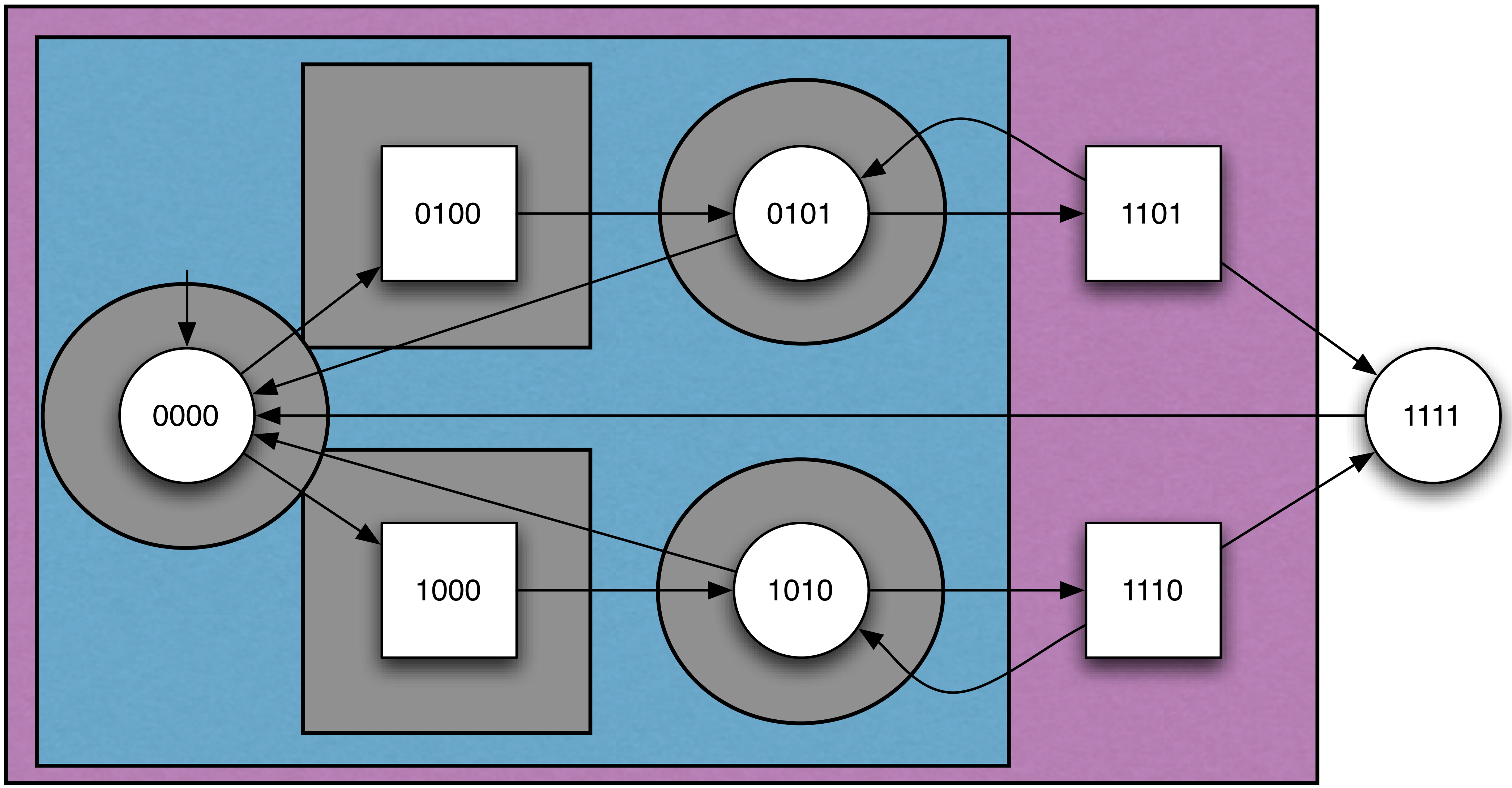
Squared positions,
all successors are red

Fixed points to solve games

Let Q be a set of safe states, the states in which Player I can force the game to stay within Q is the following greatest fixed point (computed by the previous algorithm):

$$\cup\{R \mid R = Q \cap \mathsf{CPre}_1(R)\}$$

Fixpoint for a safety game

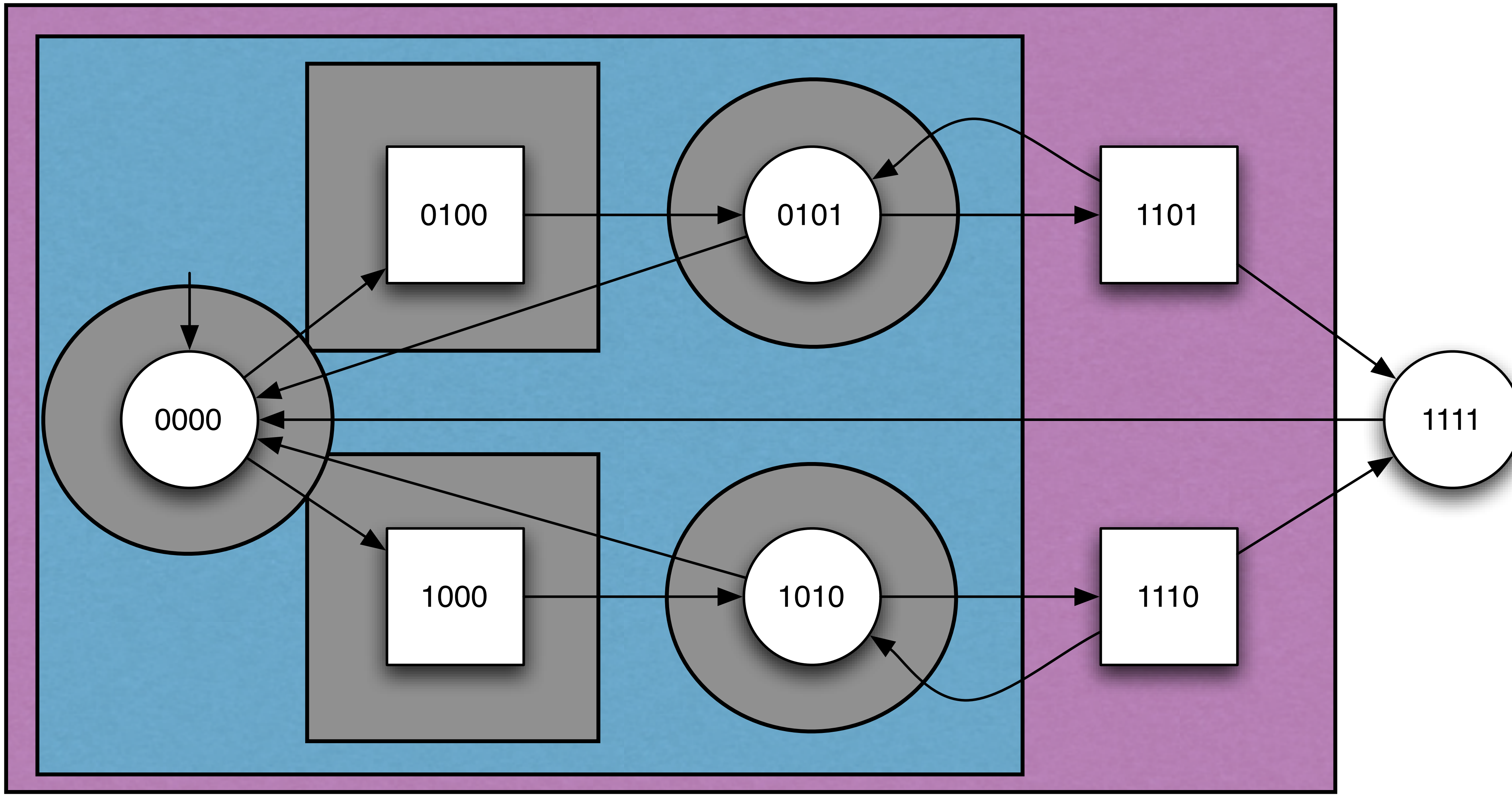


$\boxed{X_0} = (Q \setminus \{1111\}) \cap 1\mathsf{CPre}(Q)$

$\boxed{X_1} = (Q \setminus \{1111\}) \cap 1\mathsf{CPre}(X_0)$

$X_2 = \boxed{(Q \setminus \{1111\}) \cap 1\mathsf{CPre}(X_1)}$

Fixpoint for a safety game



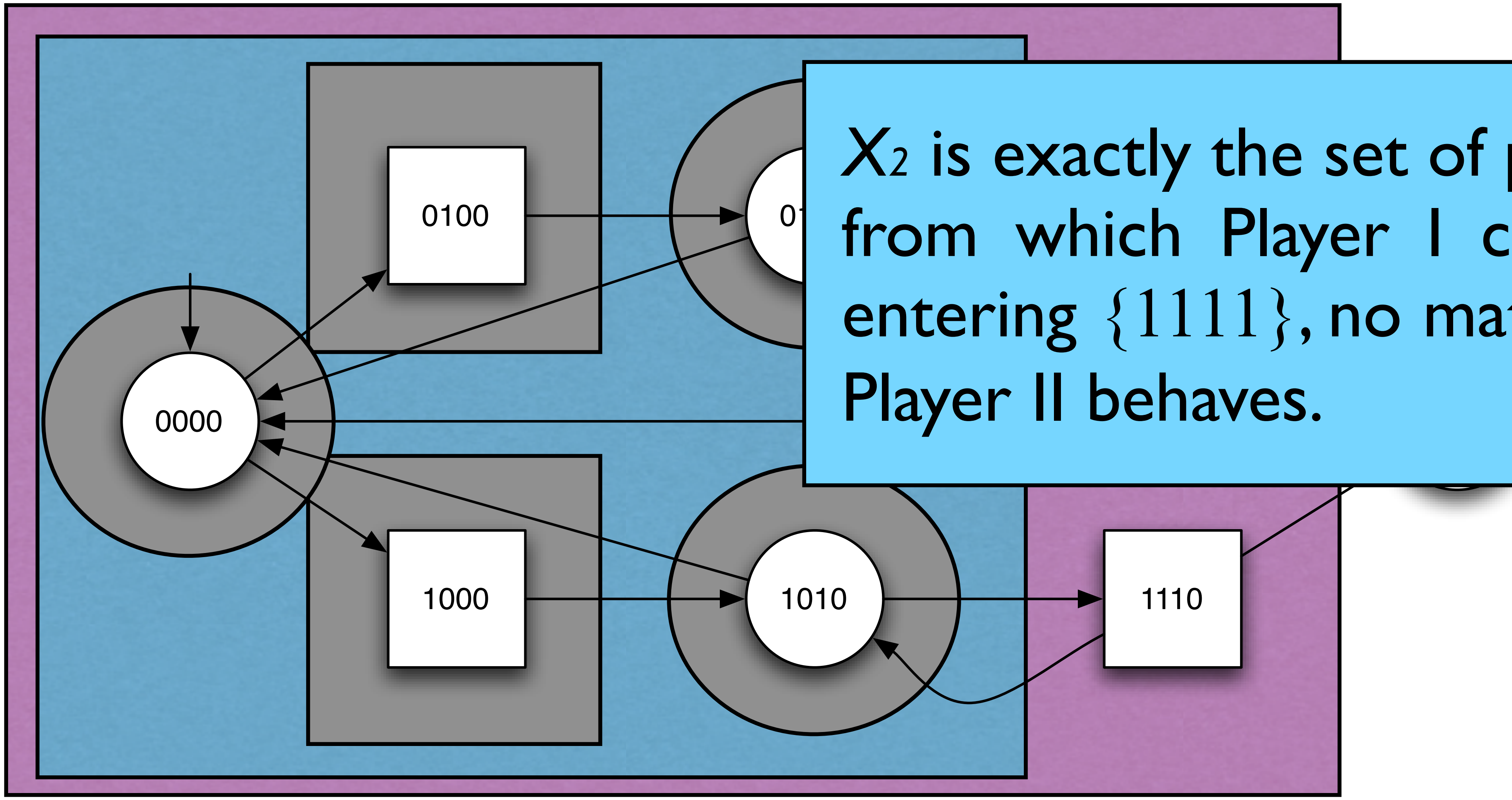
This is the greatest fixed point

$X_0 = (Q \setminus \{1111\}) \cap \mathsf{1CPre}(Q)$

$X_1 = (Q \setminus \{1111\}) \cap \mathsf{1CPre}(X_0)$

$X_2 = (Q \setminus \{1111\}) \cap \mathsf{1CPre}(X_1)$

Fixpoint for a safety game



X_2 is exactly the set of positions from which Player I can avoid entering $\{1111\}$, no matter how Player II behaves.

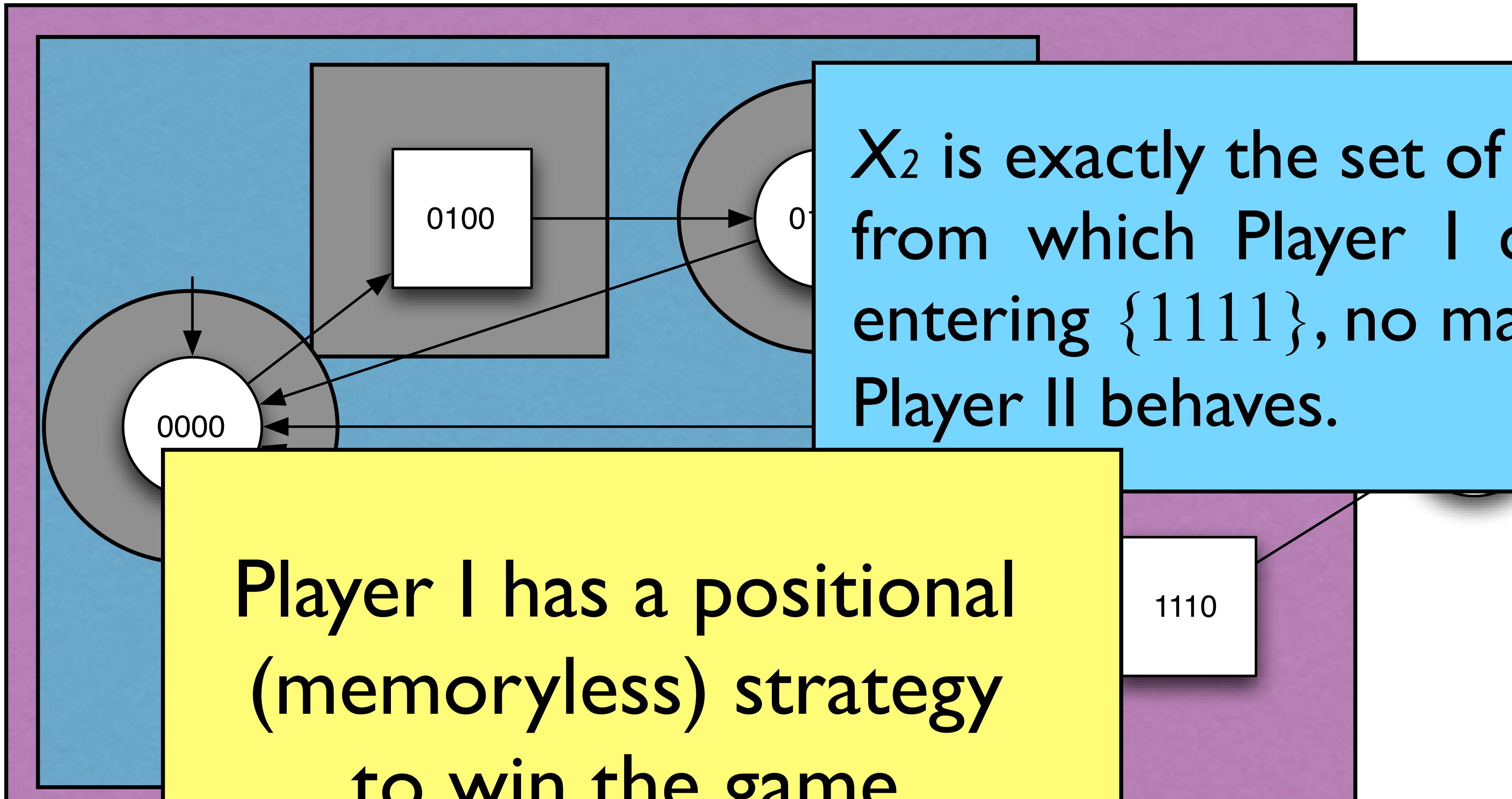
$\boxed{X_0} = (Q \setminus \{1111\}) \cap 1\mathrm{CPre}(Q)$

$\boxed{X_1} = (Q \setminus \{1111\}) \cap 1\mathrm{CPre}(X_0)$

$X_2 = \boxed{(Q \setminus \{1111\}) \cap 1\mathrm{CPre}(X_1)}$

This is the greatest fixed point

Fixpoint for a safety game



X_2 is exactly the set of positions from which Player I can avoid entering $\{1111\}$, no matter how Player II behaves.

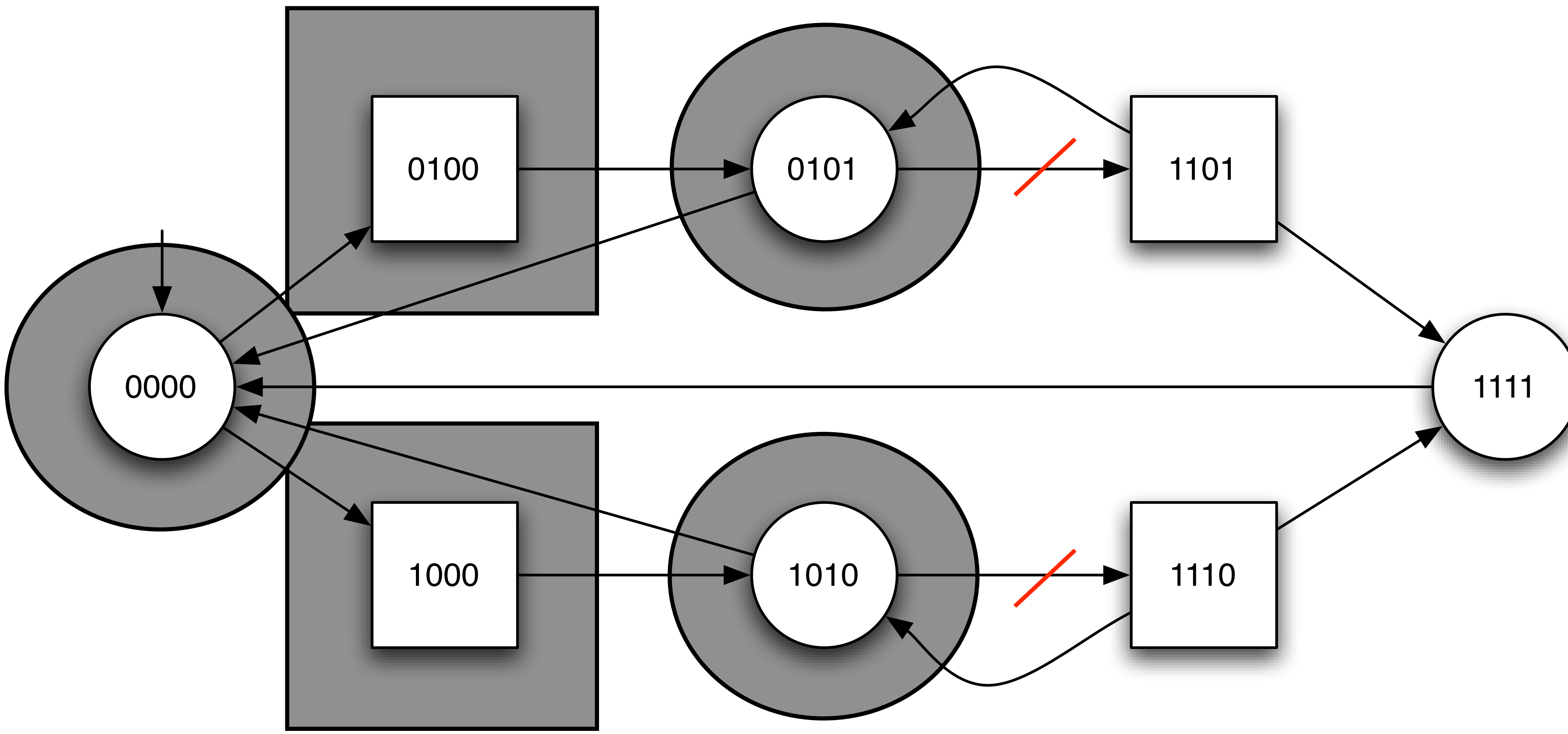
Player I has a positional (memoryless) strategy to win the game

This is the greatest fixed point

$\mathrm{Pre}(Q)$

$\boxed{X_1} = (Q \setminus \{1111\}) \cap 1\mathrm{CPre}(X_0)$

$X_2 = \boxed{(Q \setminus \{1111\}) \cap 1\mathrm{CPre}(X_1)}$



0100
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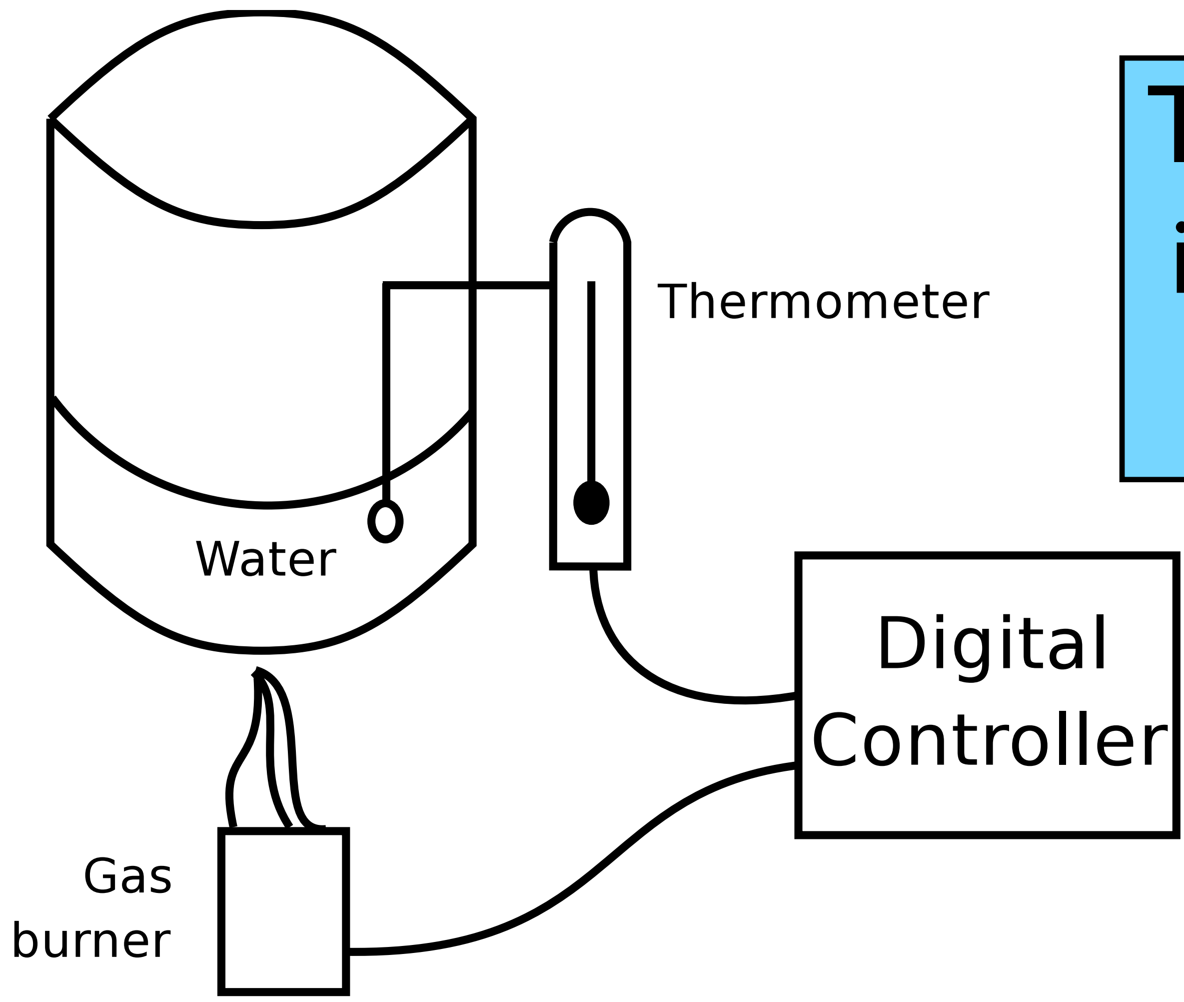
Similar fixed point algorithms exist for any **omega-regular objectives**

Games of imperfect information

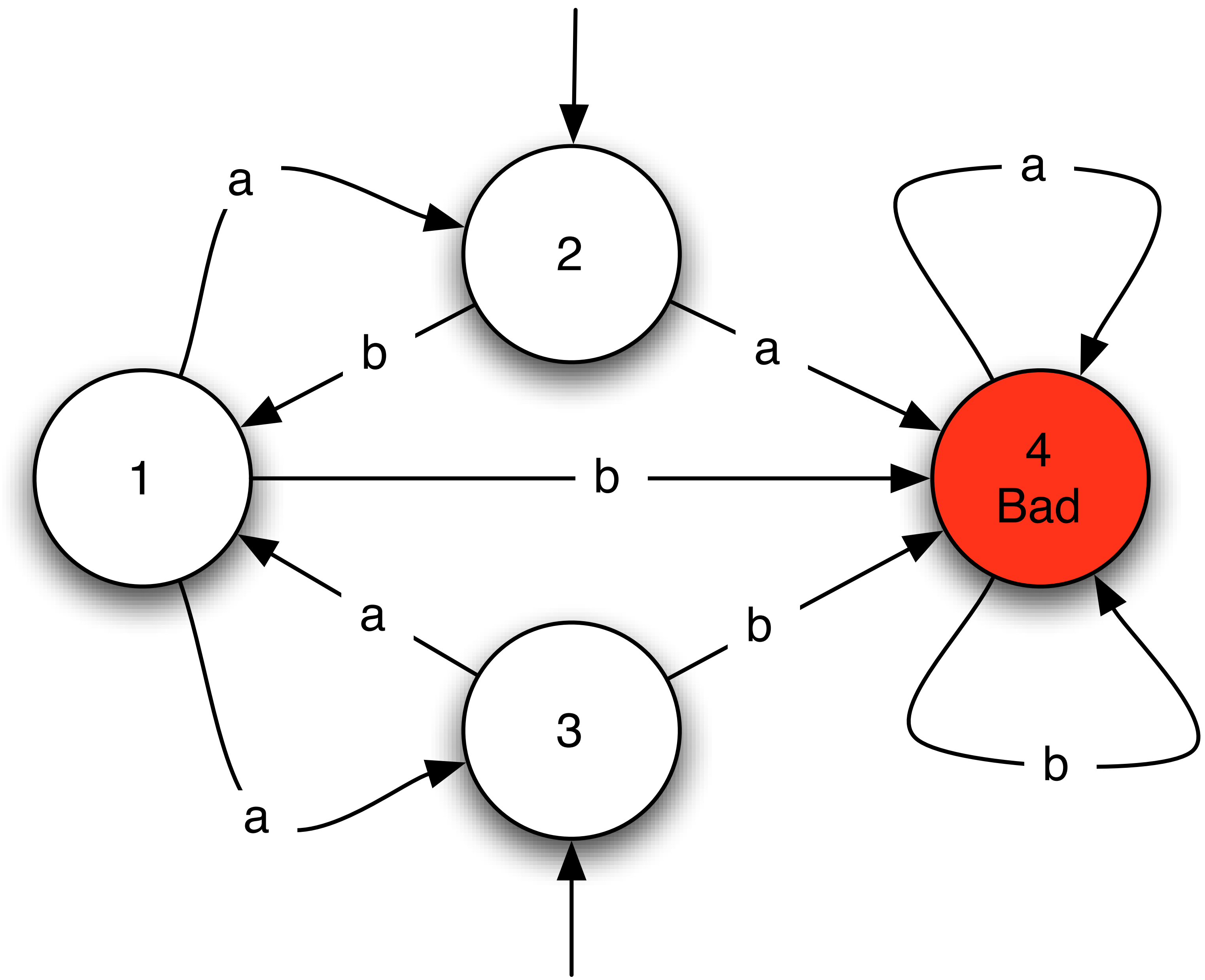
Perfect information hypothesis?

Finite precision = imperfect information

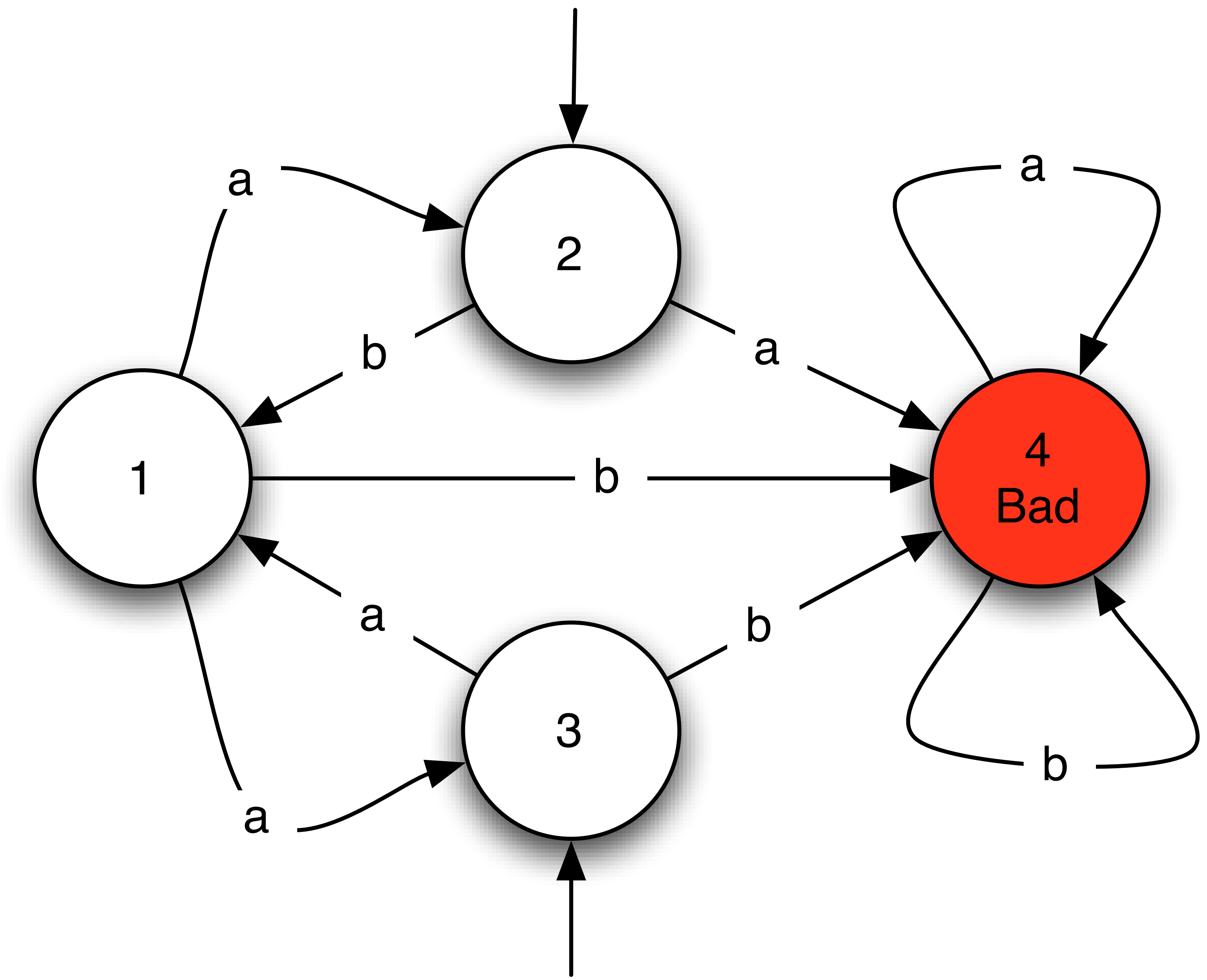


The temperature is in the interval $(c-1, c+1)$

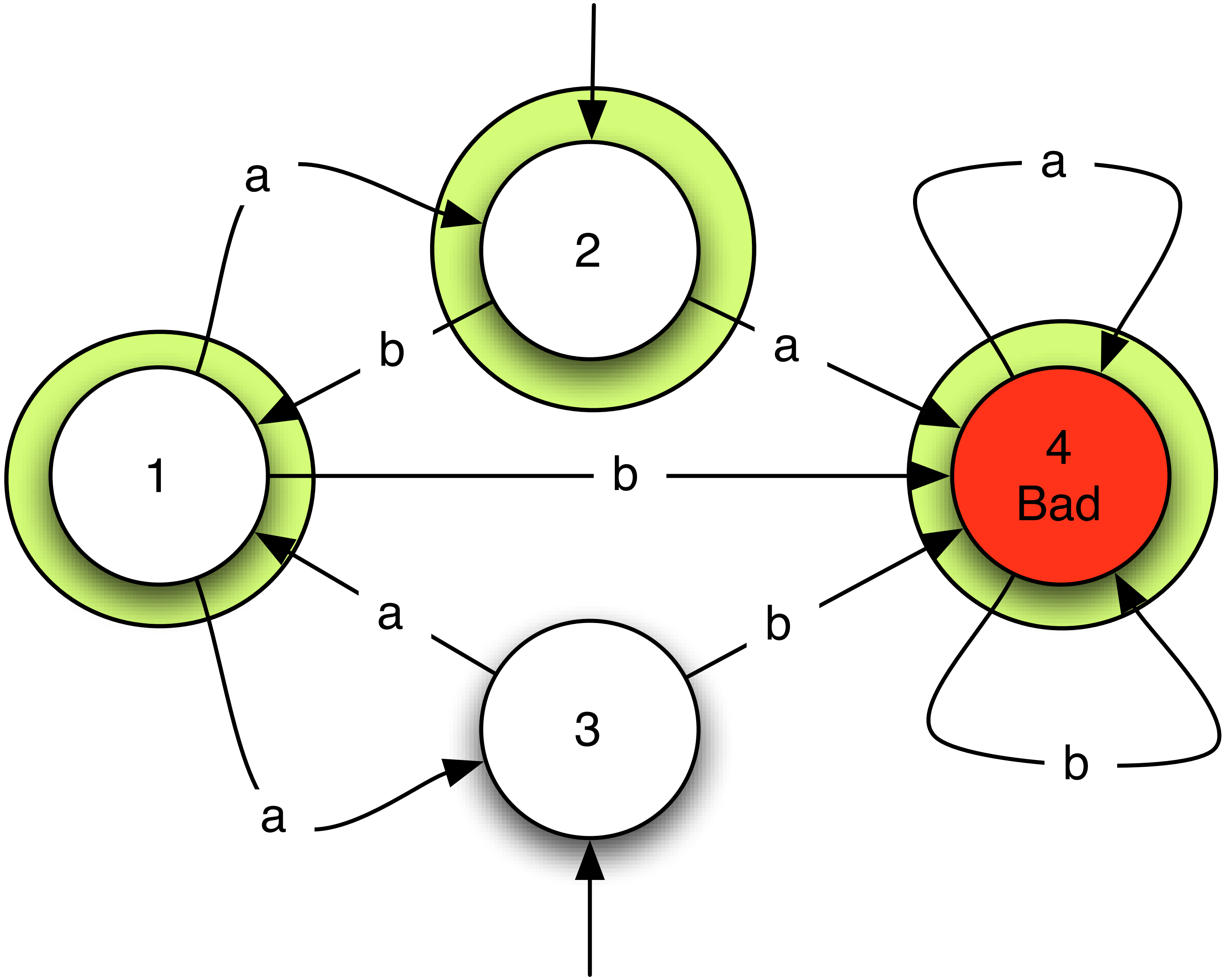
Typical hybrid system



Player 0 chooses a letter
Player 1 resolves nondeterminism

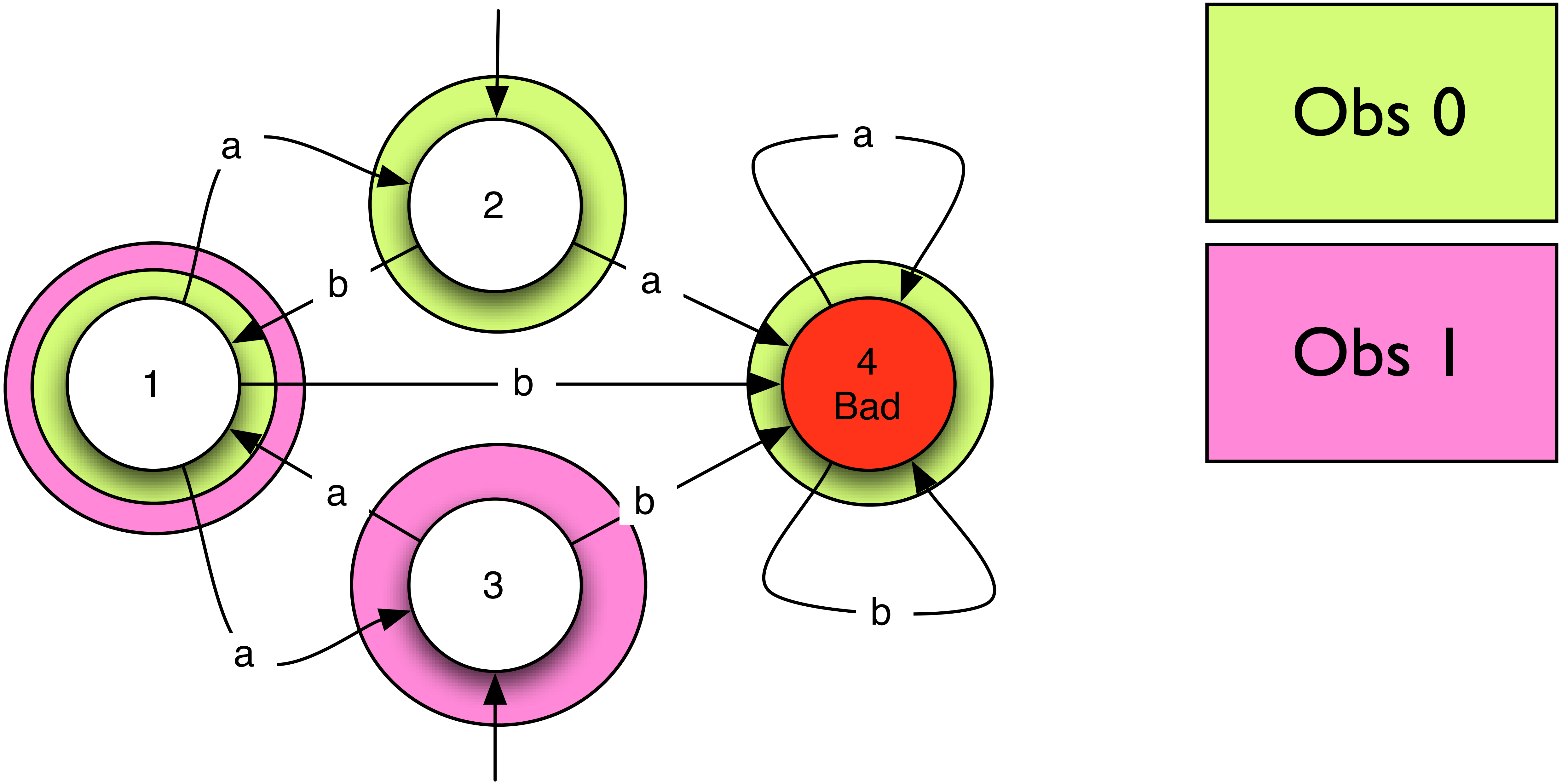


Imperfect information

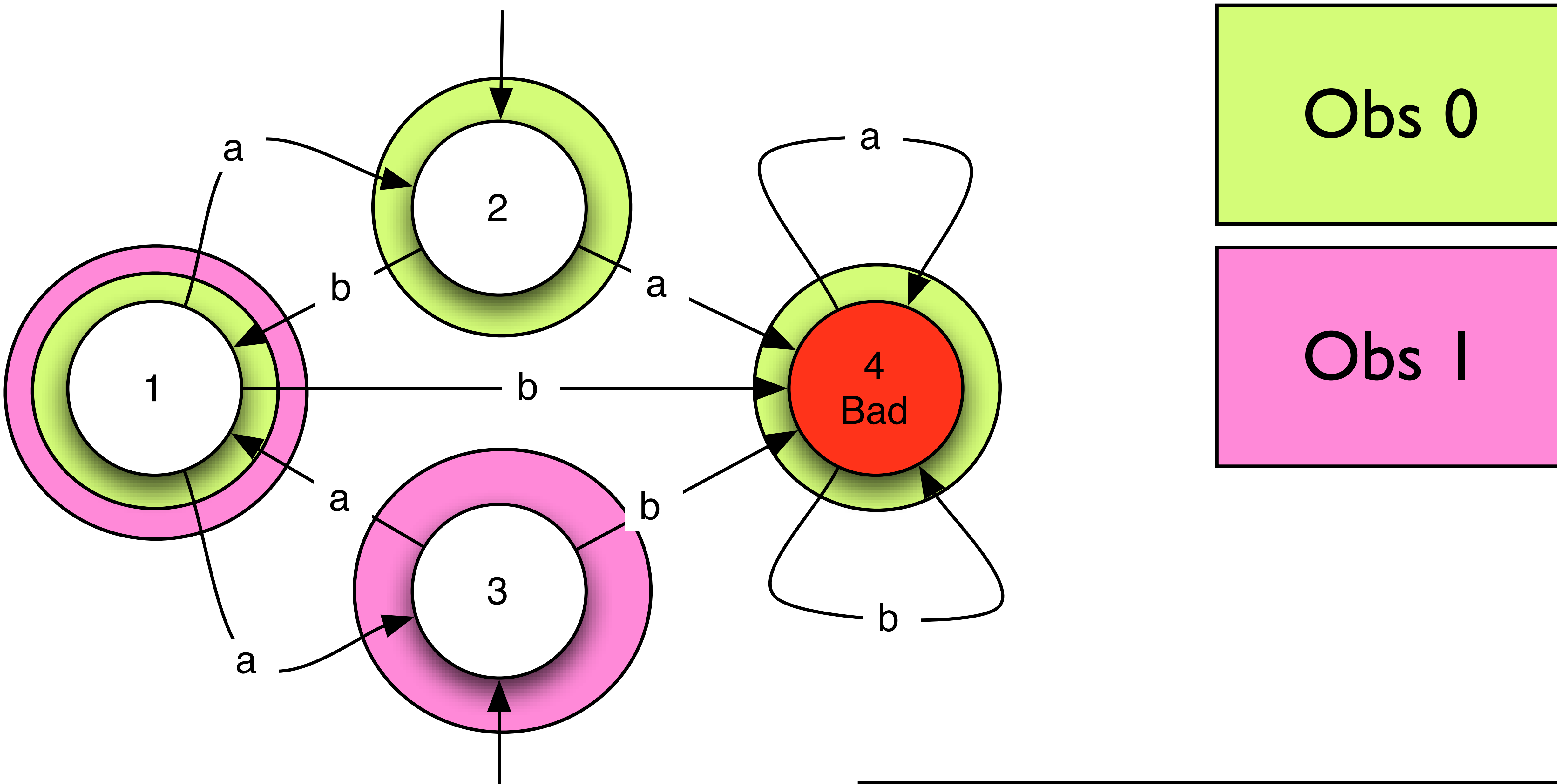


Obs 0

Imperfect information

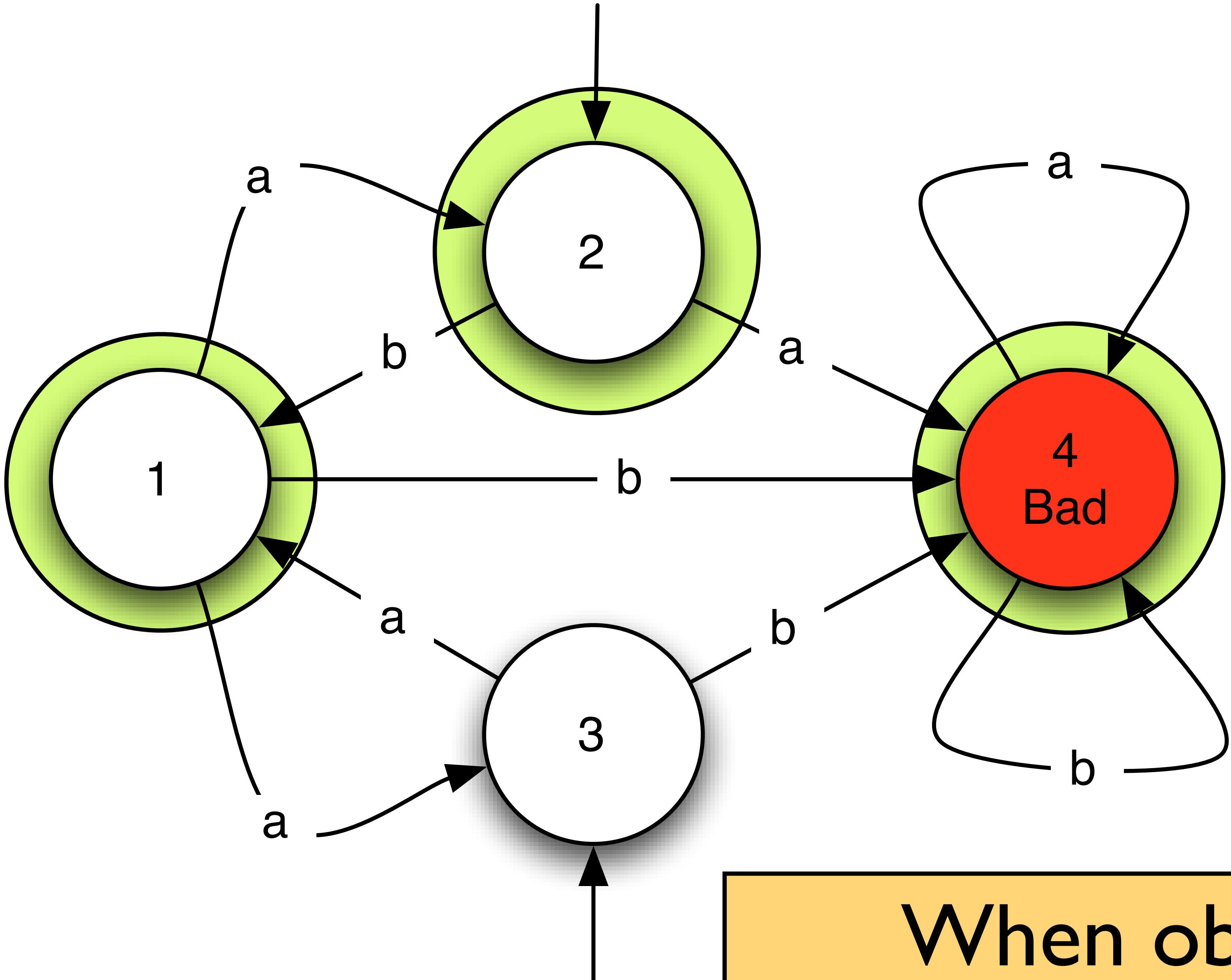


Imperfect information



Slight generalization of incomplete information

Imperfect information



Obs 0

When observing Obs 0,
there is no unique good choice:
memory is necessary

Imperfect information

- A game of *imperfect information*:

game structure + observation structure

-*Observation structure* : **(Obs,γ)** where **Obs** is a finite set of observations and **γ** maps every observation to a set of states (we require that every state has at least one observation).

-A *observation based strategy* is a function that maps every sequence $o_1 \sigma_1 o_2 \ldots o_n$ to a letter in Σ.

Our objective is to find an algorithm to construct **observation based strategies** that avoid Bad.

Notation: a game structure of imperfect information is a tuple $(S, S_0, \Sigma, \rightarrow, Obs, \gamma)$.

...ation structure

...where **Obs** is a finite set of observations and **γ** maps every observation to a set of states (we require that every state has at least one observation).

-A *observation based strategy* is a function that maps every sequence $o_1 \sigma_1 o_2 \ldots o_n$ to a letter in Σ.

Our objective is to find an algorithm to construct **observation based strategies** that avoid Bad.

Notation: a game structure of imperfect information is a tup

Those games generalize games of *perfect information* where **Obs=S** and γ is the identity function

ation structure

s a finite

set of ... ation to

a set o ... as at

least o

-A *observation based strategy* is a function that maps every sequence $o_1\sigma_1o_2 \ldots o_n$ to a letter in Σ.

Our objective is to find an algorithm to construct **observation based strategies** that avoid Bad.

Notation: a game structure of imperfect information is a tup

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Those games generalize games of p

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Those games generalize games of *incomplete information*: in that case **Obs partitions** the state space *S*. [Rei84]

-A *observa*

s

every sequ

Our objective is to find an algorithm to construct **observation based strategies** that avoid Bad.

- To solve games of perfect information :
  - (elegant) fixed point algorithms using a controllable predecessor operator
- To solve games of imperfect information
  - [Reif84] builds a game of perfect information using a knowledge-based subset construction and then solve this games using classical techniques

- To solve games of perfect information :
  - (ele
    cor
- To so
  - [Reif84] builds a game of perfect information using a knowledge-based subset construction and then solve this games using classical techniques

After a finite prefix of a game, Player I has a partial knowledge of the current state of the game : **a set of states**

- To solve games of perfect information :
  - (ele
    cor

After a finite prefix of a game, Player I has a partial knowledge of the current state of the game : **a set of states**

We propose here a new solution that avoids the **preliminary** explicit subset construction.

ne of perfect
knowledge-based
subset construction and then solve this games using classical techniques

We define a *controllable predecessor* operator for a ***set of sets of states*** *q*

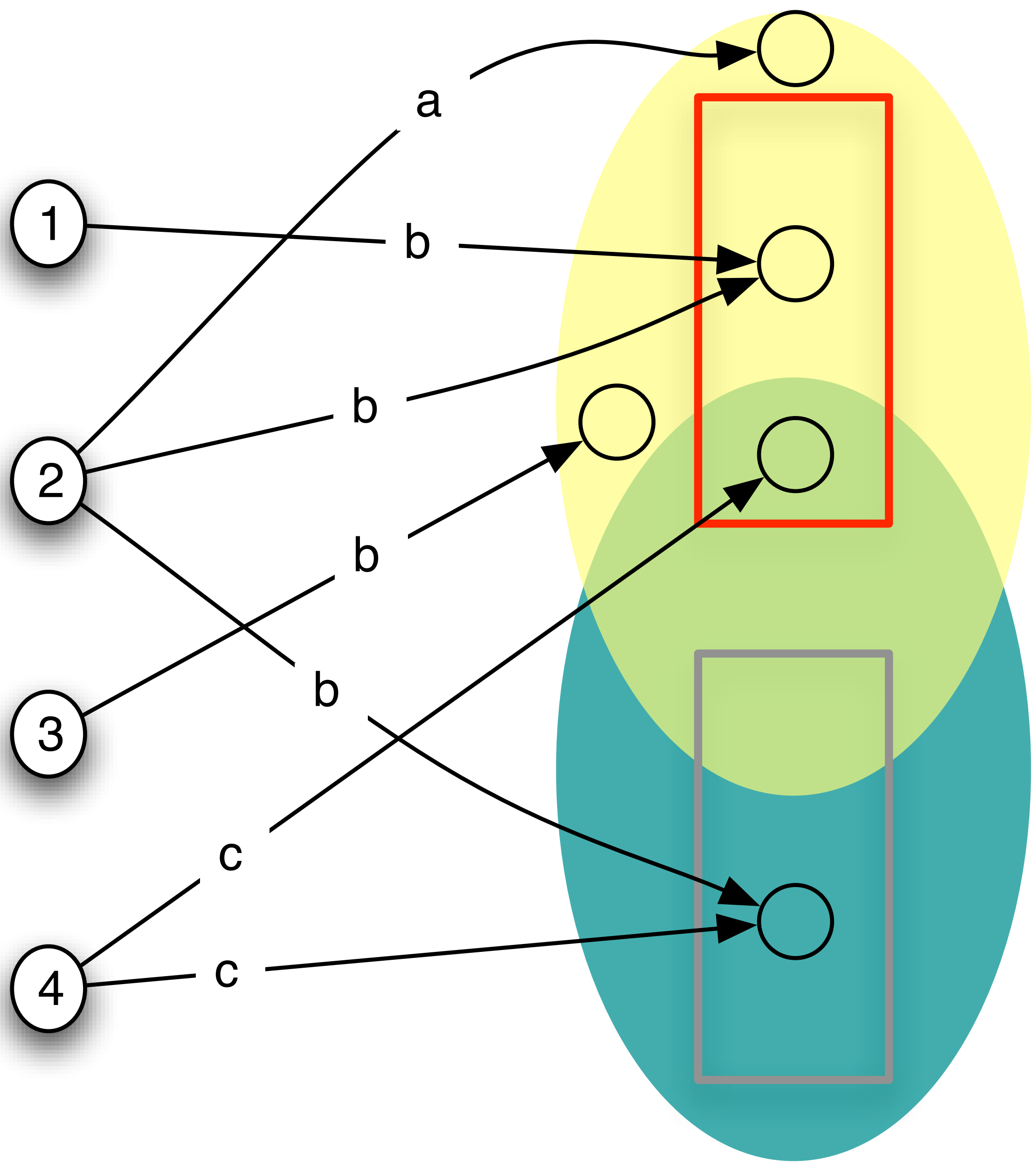
$$\mathsf{CPre}(q) = \{s \subseteq \overline{\mathsf{Bad}} \mid \exists \sigma \in \Sigma \cdot \forall \mathsf{obs} \in \mathsf{Obs} \cdot \exists s' \in q : \mathsf{Post}_\sigma(s) \cap \gamma(\mathsf{obs}) \subseteq s'\}$$

(i) *s* does not intersect with **Bad**,

(ii) there exists σ s.t. the set of possible successors of s by σ is covered by *q*

(a) no matter how the adversary resolves non-determinism,

(b) no matter the compatible observation **Obs**

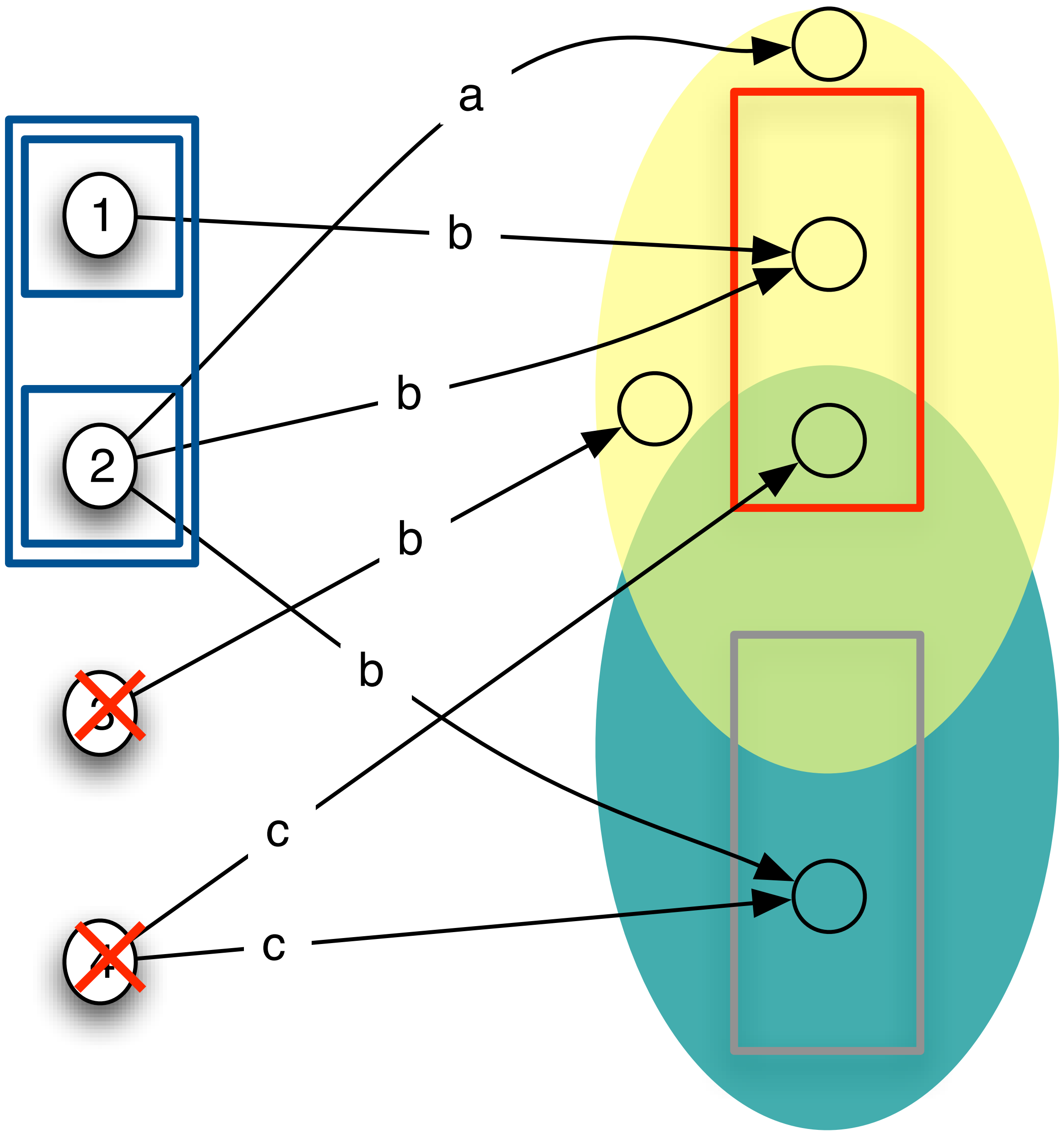


$q = \{A, B\}$

Obs 1

Obs 2

Example



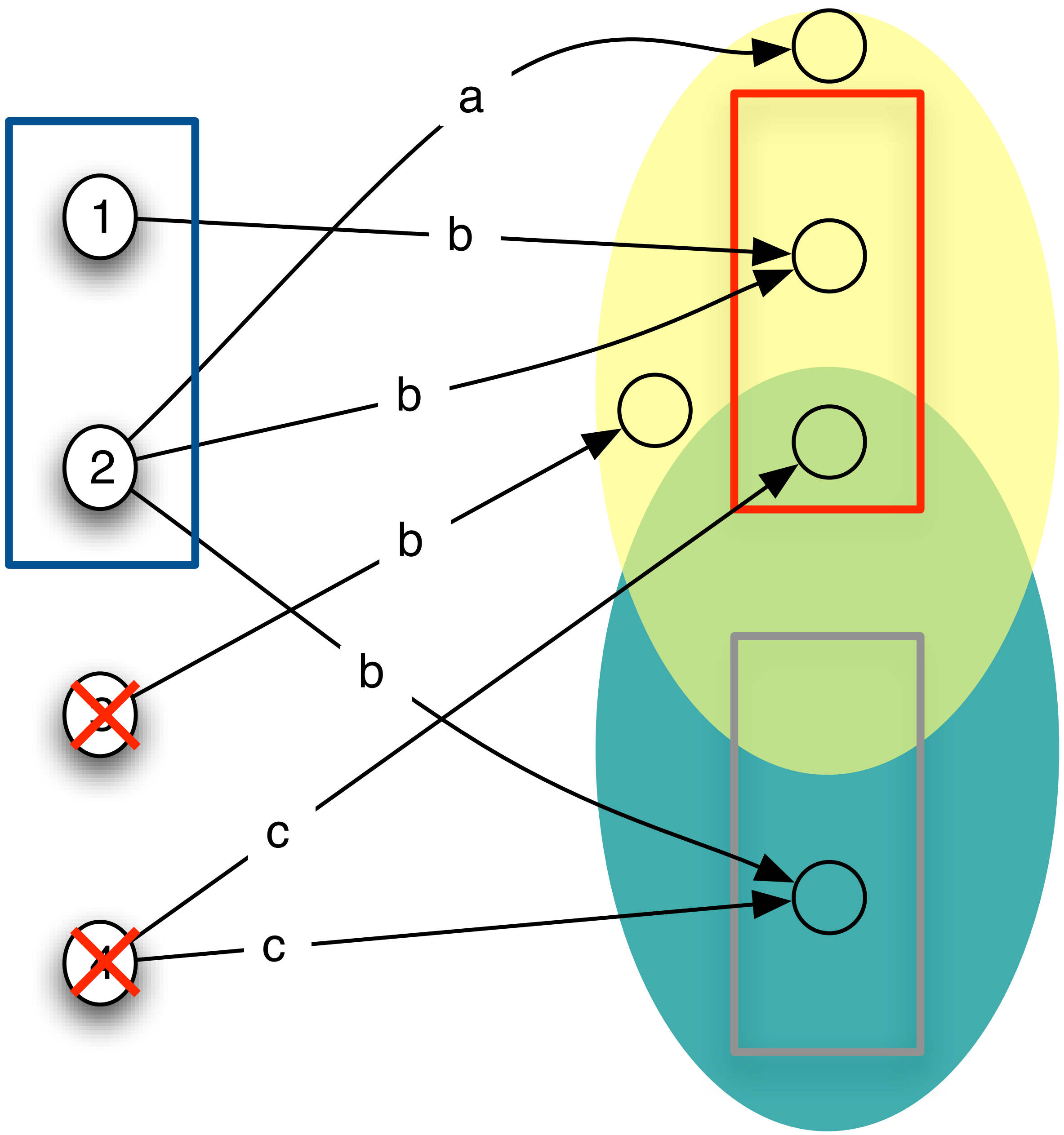
q ={A, B}

Obs 1

Obs 2

Cpre({A,B})= Blue sets

If there is a strategy for set A,
there is a strategy for any B included in A



It is enough to keep only the **maximal sets**

$$\mathsf{CPre}(q) = [\{s \subseteq \overline{\mathsf{Bad}} \mid \exists \sigma \in \Sigma \cdot \forall \mathsf{obs} \in \mathsf{Obs} \cdot \exists s' \in q : \mathsf{Post}_\sigma(s) \cap \gamma(\mathsf{obs}) \subseteq s'\}]$$

Antichains

Definition 4 [Antichain of sets of states] An *antichain* on the partially ordered set $\langle 2^S, \subseteq \rangle$ is a set $q \subseteq 2^S$ such that for any $A, B \in q$ we have $A \not\subseteq B$.

Let us call L the set of antichains on S.

Definition 5 [$\sqsubseteq$] Let $q, q' \in 2^{2^S}$ and define $q \sqsubseteq q'$ if and only if

$$\forall A \in q : \exists A' \in q' : A \subseteq A'$$

$$\mathbf{lub}: \ q_1 \sqcup q_2 = \lceil \{ s \mid s \in q_1 \vee s \in q_2 \} \rceil$$

$$\mathbf{glb}: \ q_1 \sqcap q_2 = \lceil \{ s_1 \cap s_2 \mid s_1 \in q_1 \wedge s_2 \in q_2 \} \rceil$$

The minimal element is $\emptyset$, the maximal element $\{S\}$.

$\langle L, \sqsubseteq \rangle$ is a complete lattice.

$$\mathsf{CPre}(q) = \lceil\{s \subseteq \overline{\mathsf{Bad}} \mid \exists \sigma \in \Sigma \cdot \forall \mathsf{obs} \in \mathsf{Obs} \cdot \exists s' \in q : \mathsf{Post}_\sigma(s) \cap \gamma(\mathsf{obs}) \subseteq s'\}\rceil$$

- CPre is a **monotone** function over the lattice of antichains
- CPre has a *least* and a *greatest* fixed point

Advantage : we only keep the **needed information** to find a strategy

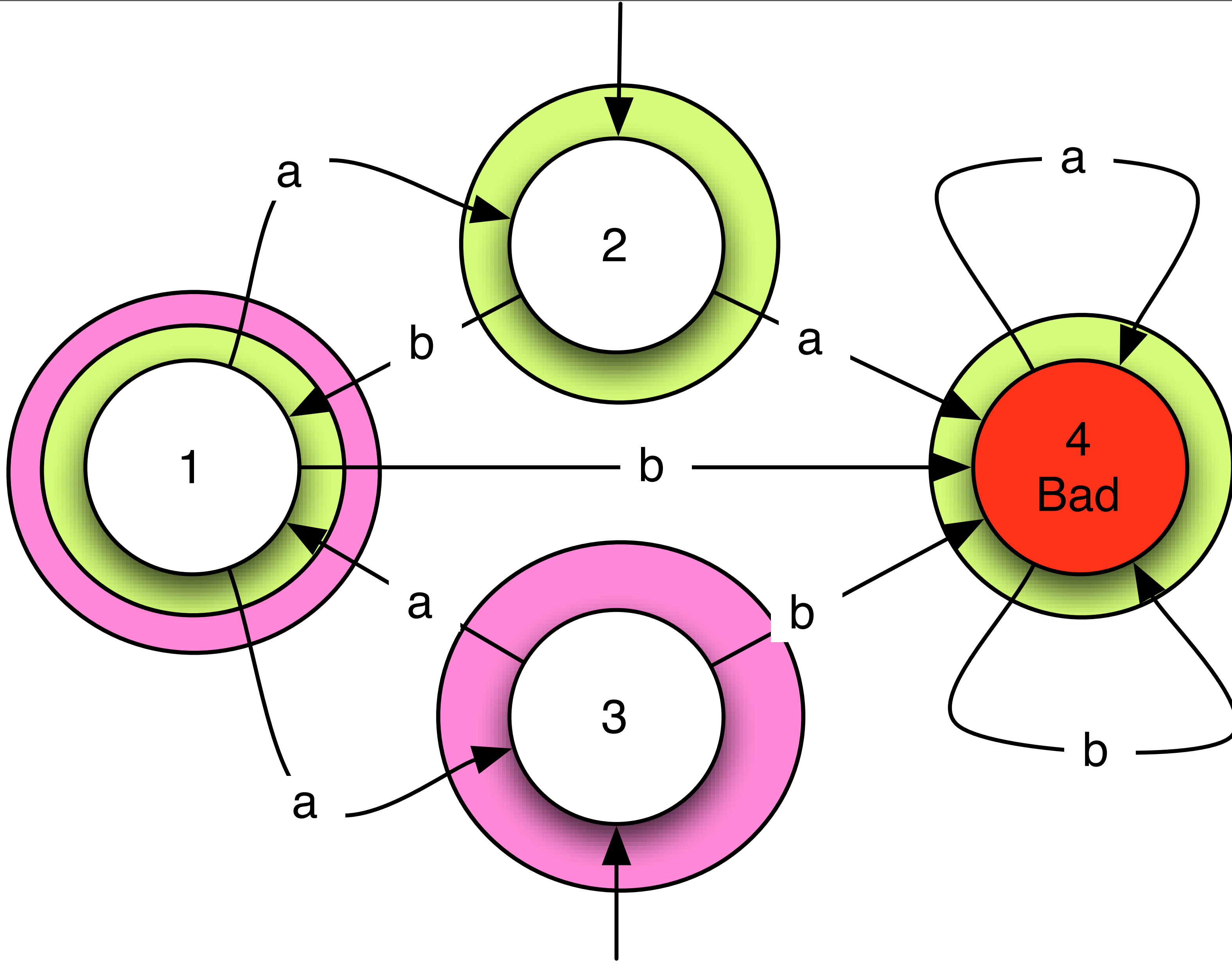
Main theorem

Let $G = \langle S, S_0, \Sigma, \rightarrow, \mathsf{Obs}, \gamma \rangle$

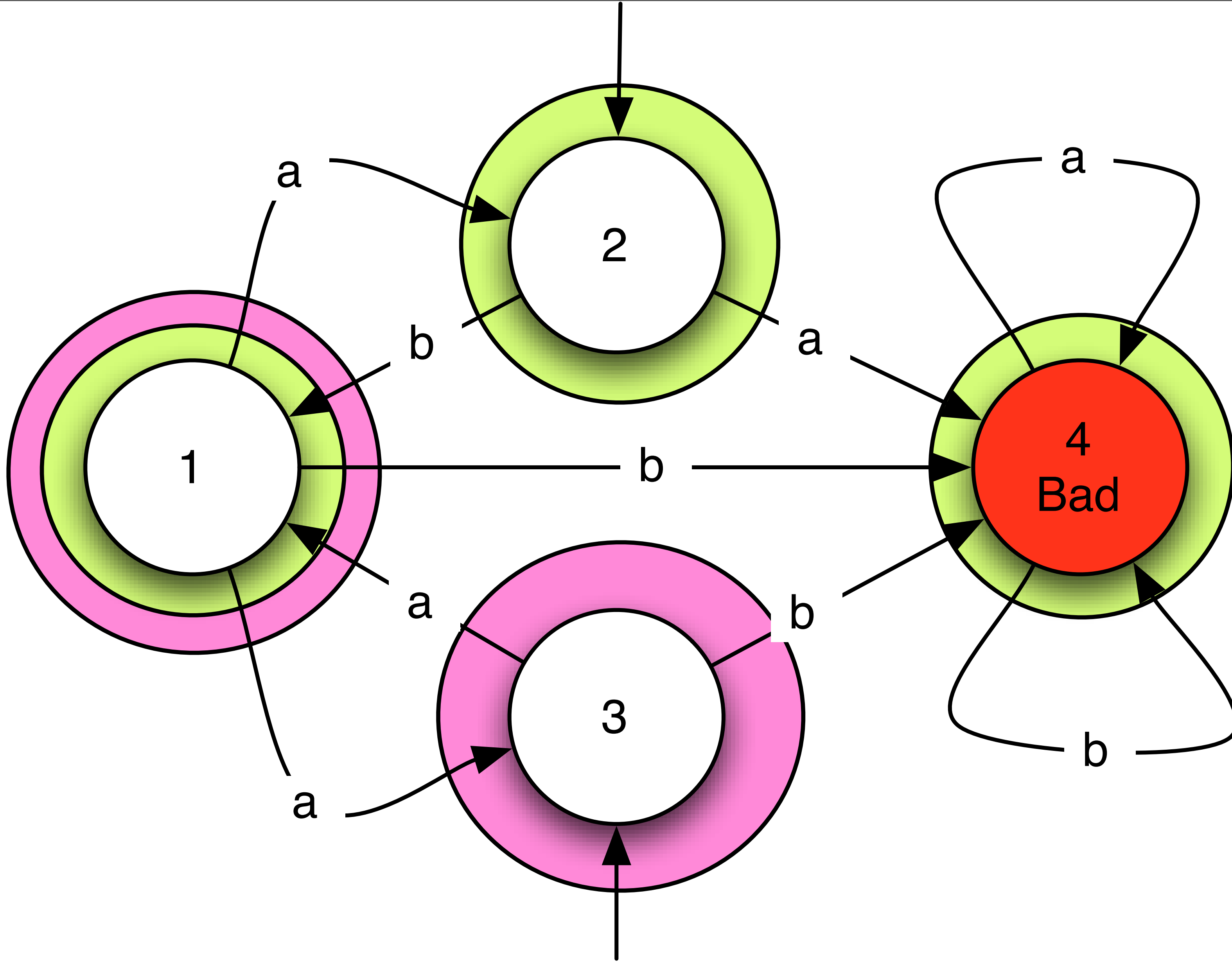
be a two-player game of imperfect information. Player 1 has a winning observation based strategy to avoid Bad, **iff**

$$\{S_0 \cap \gamma(\mathsf{obs}) \mid \mathsf{obs} \in \mathsf{Obs}\} \sqsubseteq \bigsqcup \{q \mid q = \mathsf{CPre}(q)\}.$$

We can extract a strategy from the fixed point

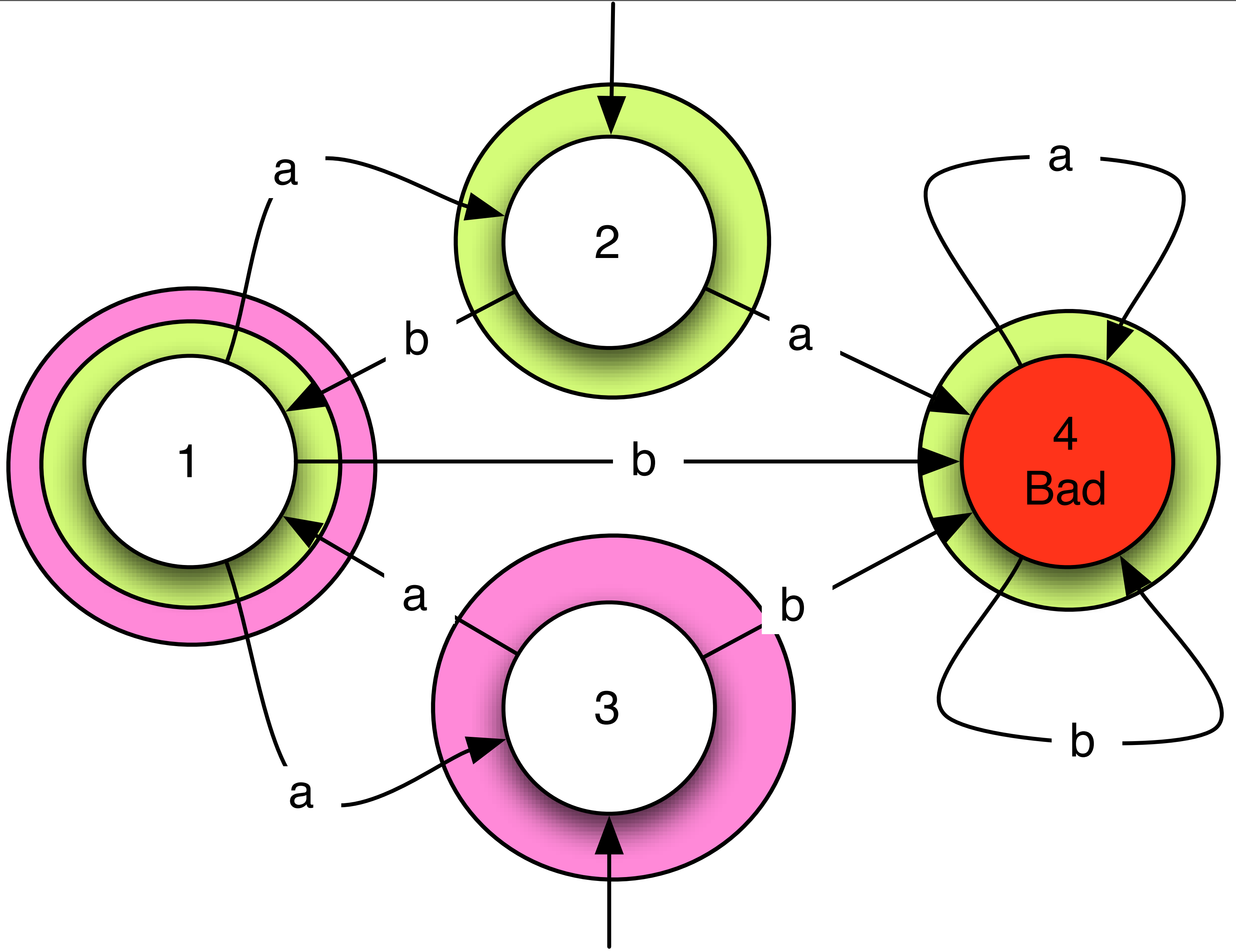


Does Player 0 have an observation based strategy to avoid Bad ?



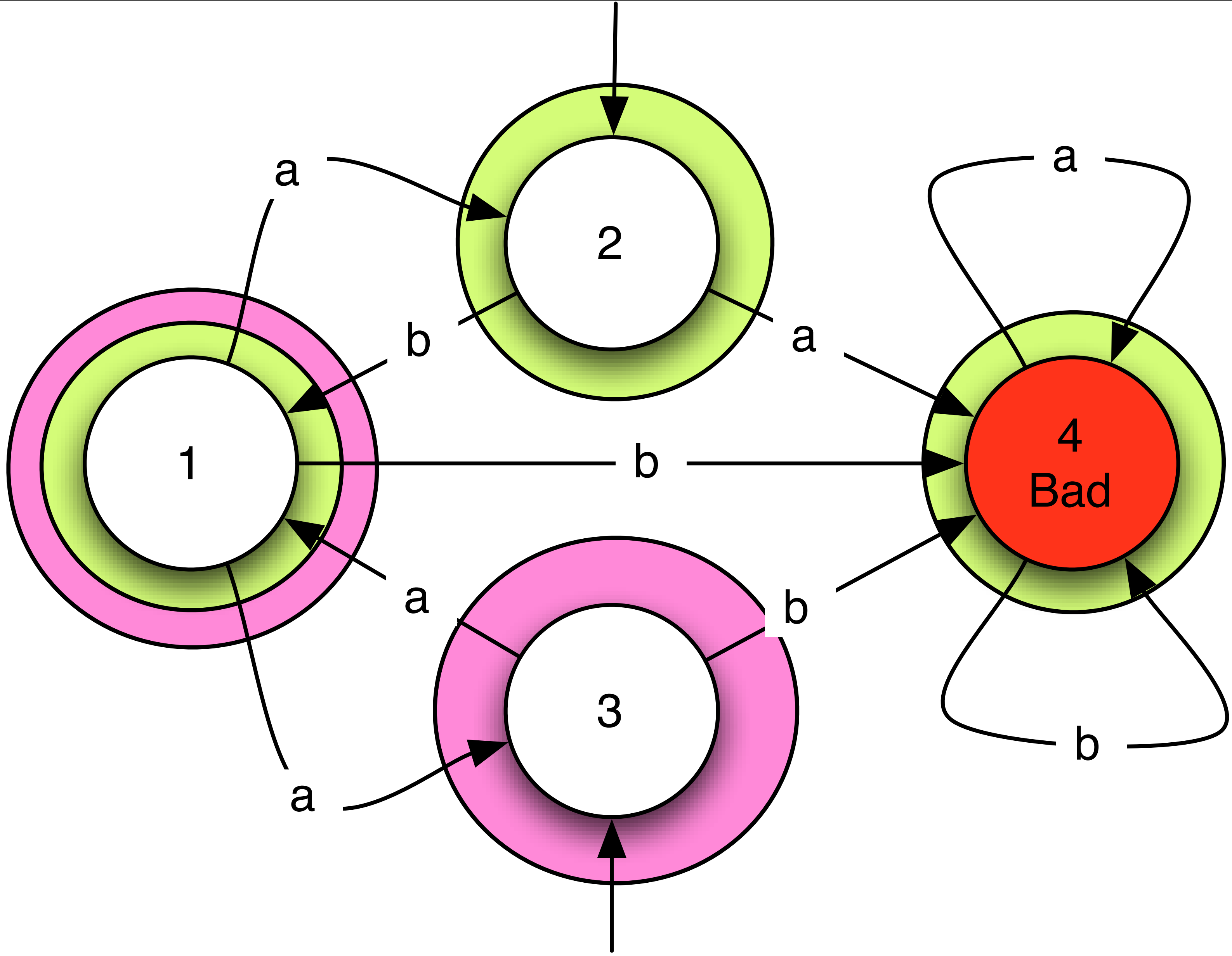
Does Player 0 have an observation based strategy to avoid Bad ?

Let us compute the *gfp* of CPre over L.



$q_0 = \top$

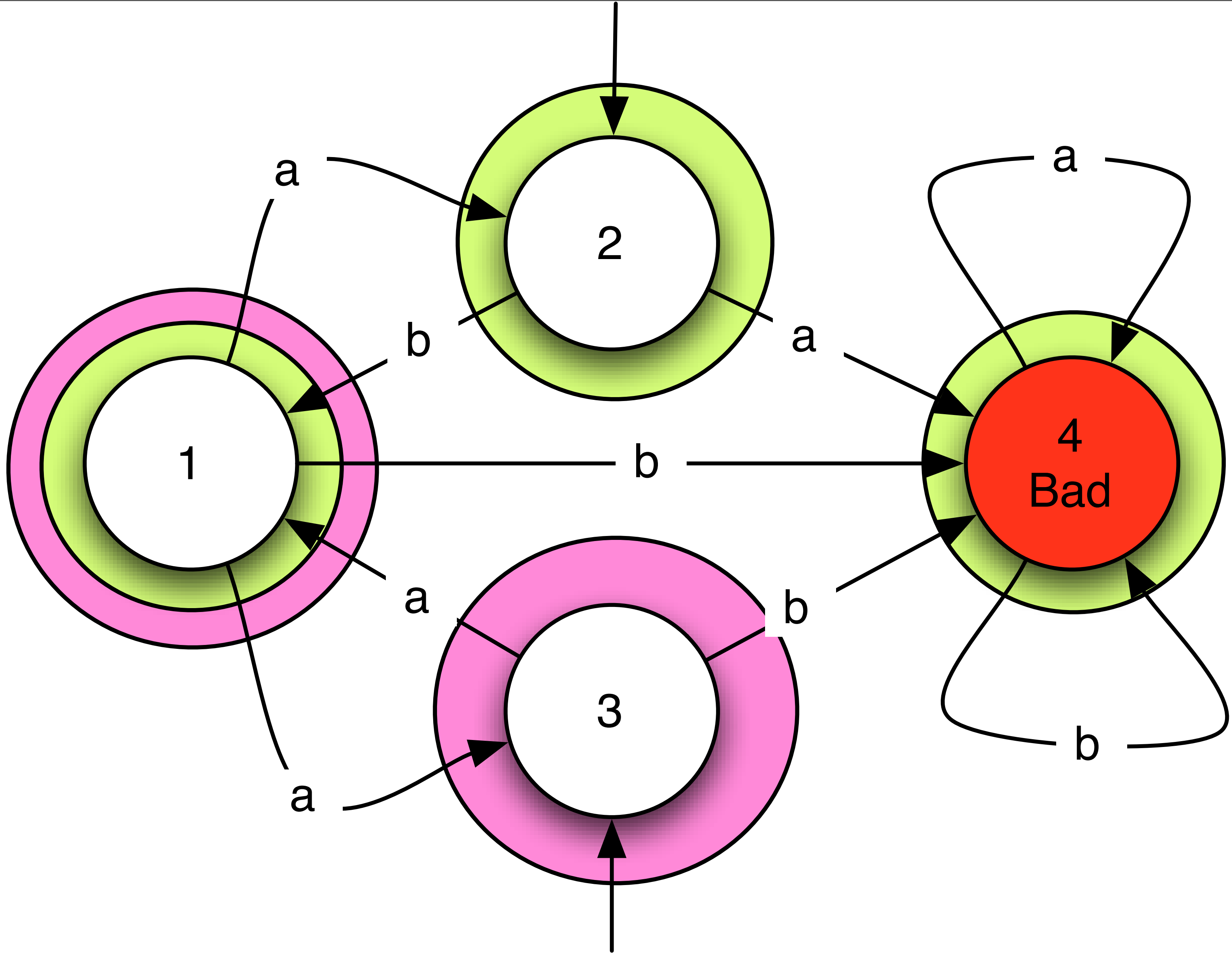
$q_1 = \{\{1, 2, 3\}_{a,b}\}$



$$q_0 = \top$$

$$q_1 = \{\{1, 2, 3\}_{a,b}\}$$

$$q_2 = \mathsf{CPre}(\{\{1, 2, 3\}\})$$



$$q_0 = \top$$
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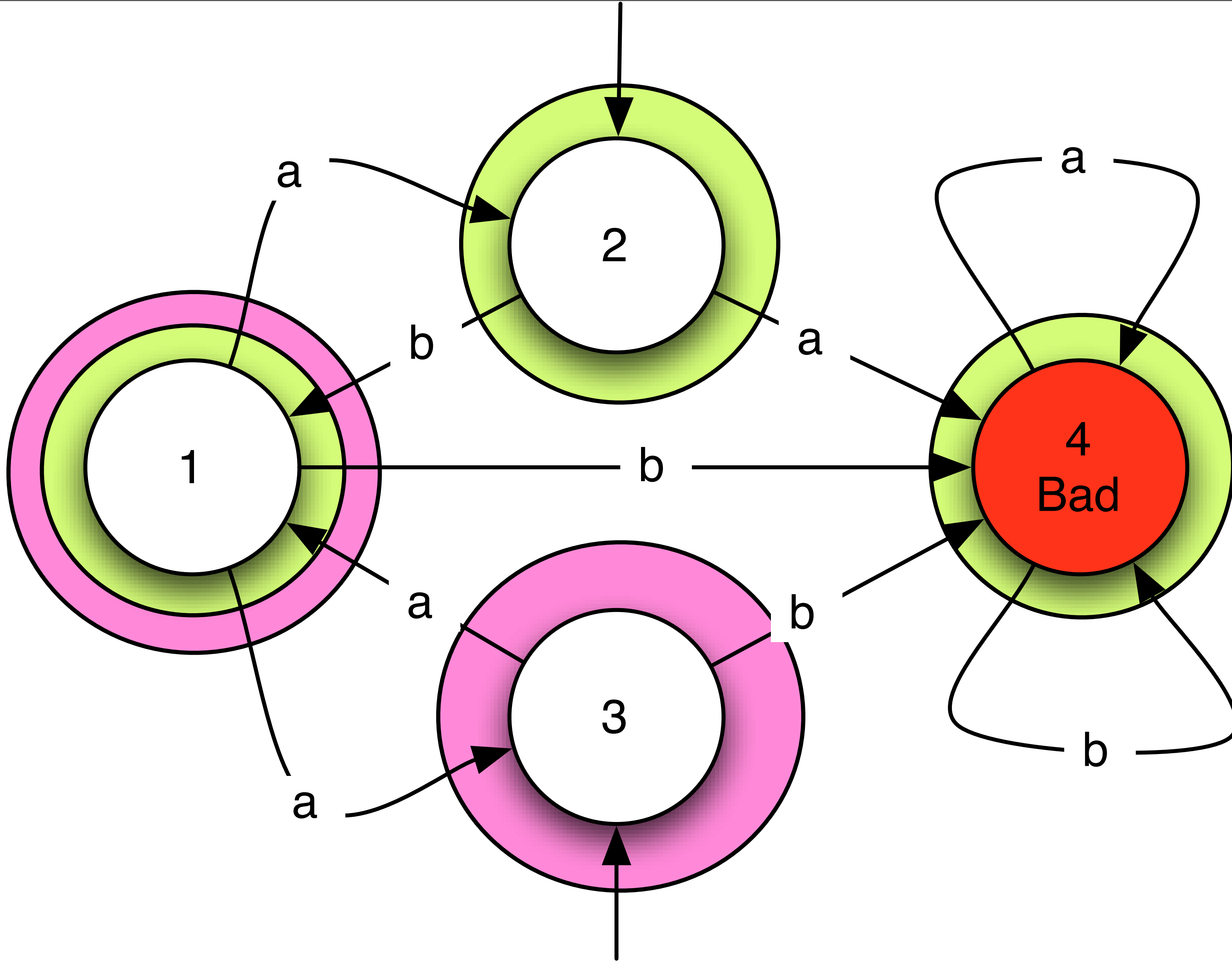
$$q_2 = \mathsf{CPre}(\{\{1,2,3\}\})$$
$$= \{\{2\}_b, \{1,3\}_a\}$$



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Indeed,

$\mathsf{Post}_a(\{1,3\}) \cap \{1,2,4\} \subseteq \{1,2,3\}$

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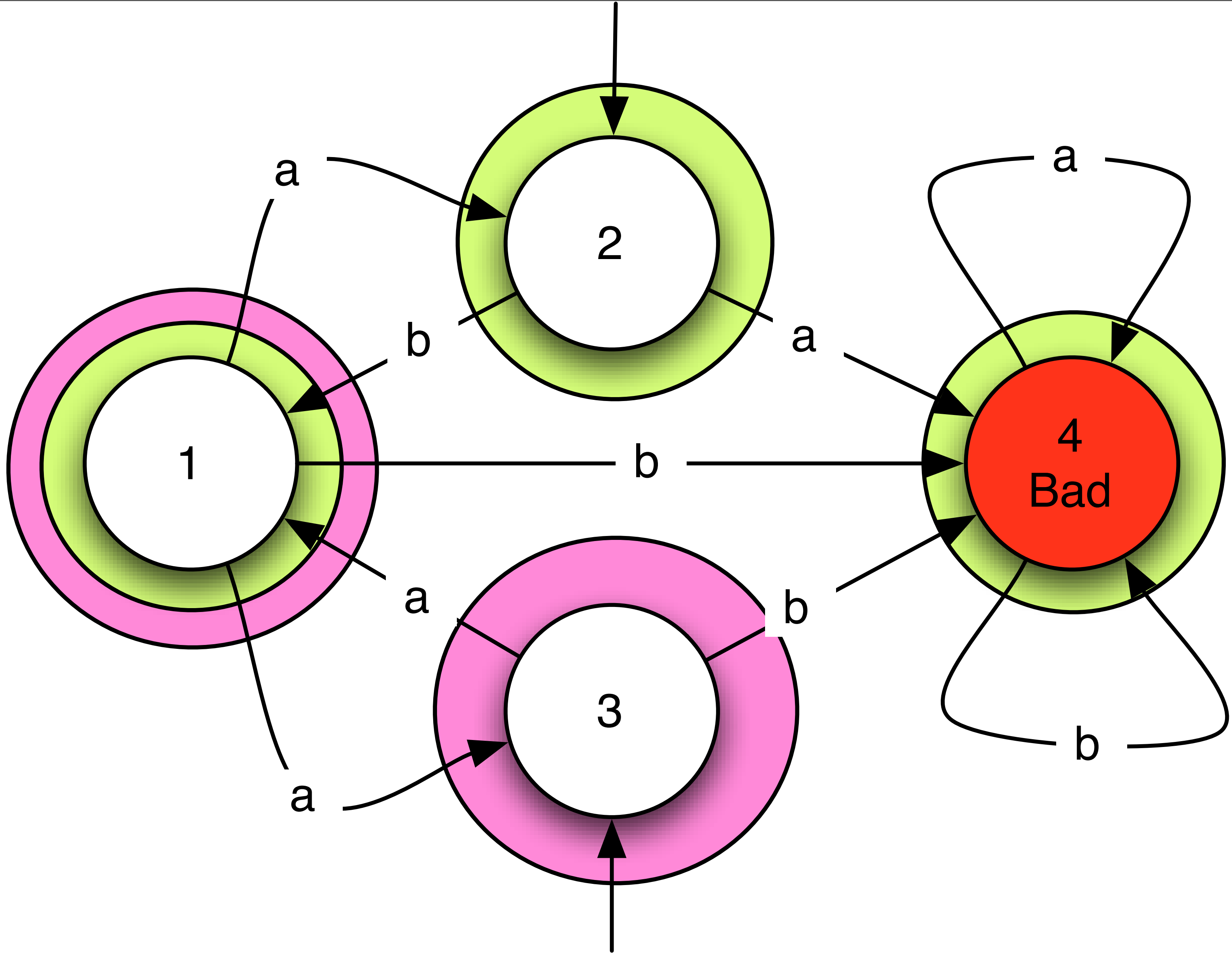
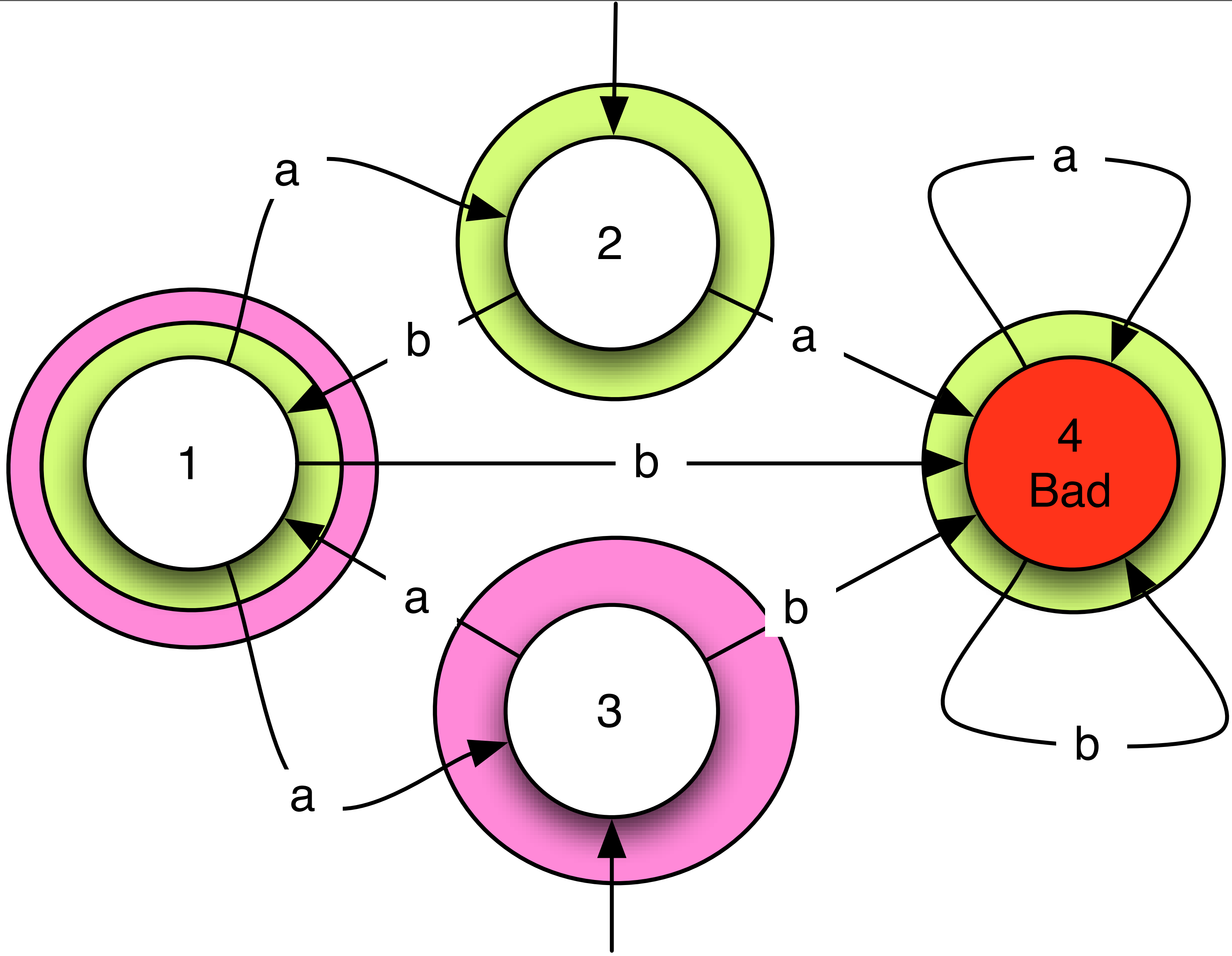


$$q_0 = \top$$

$$q_1 = \{\{1, 2, 3\}_{a,b}\}$$

$$q_2 = \{\{2\}_b, \{1, 3\}_a\}$$

$$q_3 = \mathsf{CPre}(\{\{2\}, \{1, 3\}\})$$



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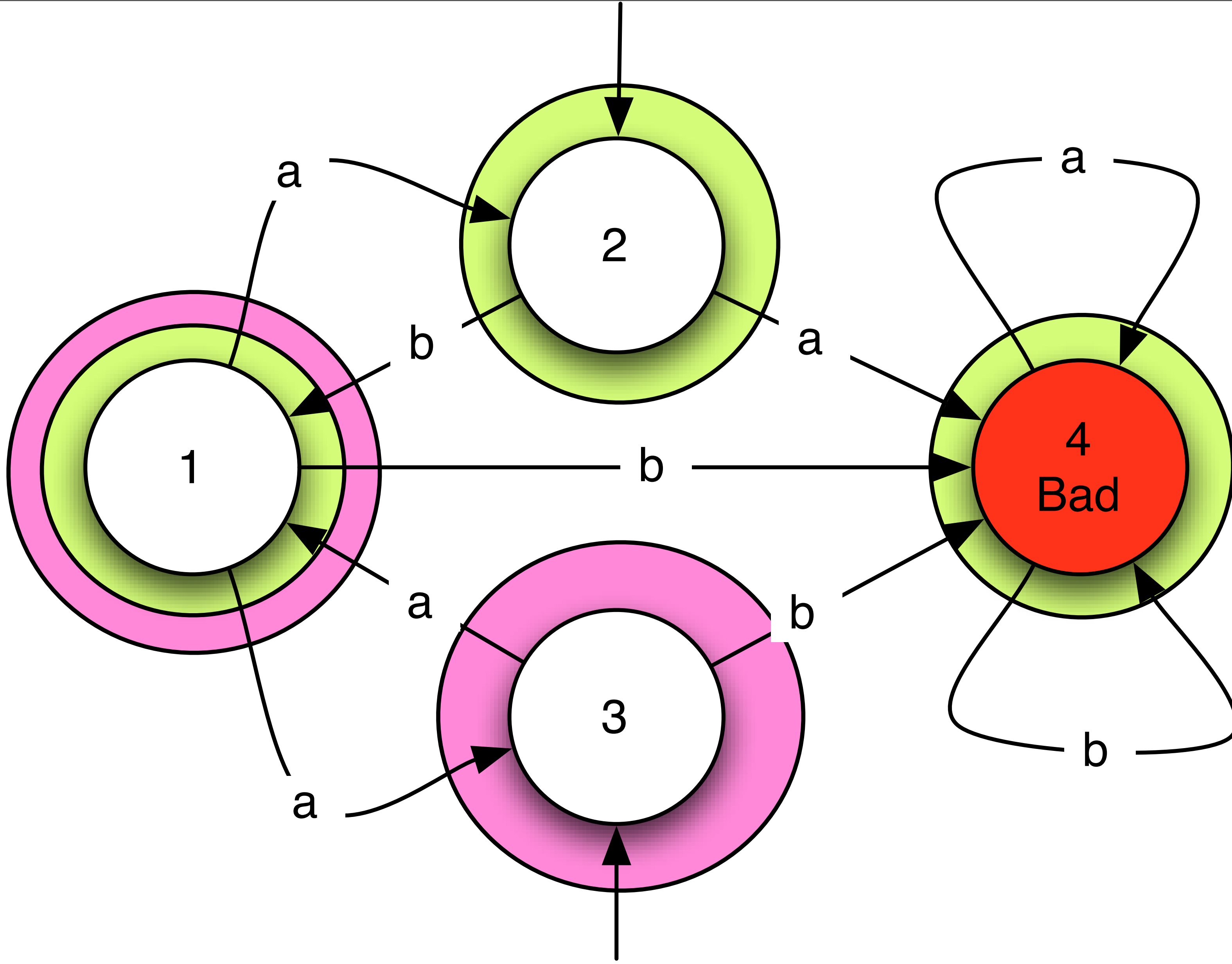

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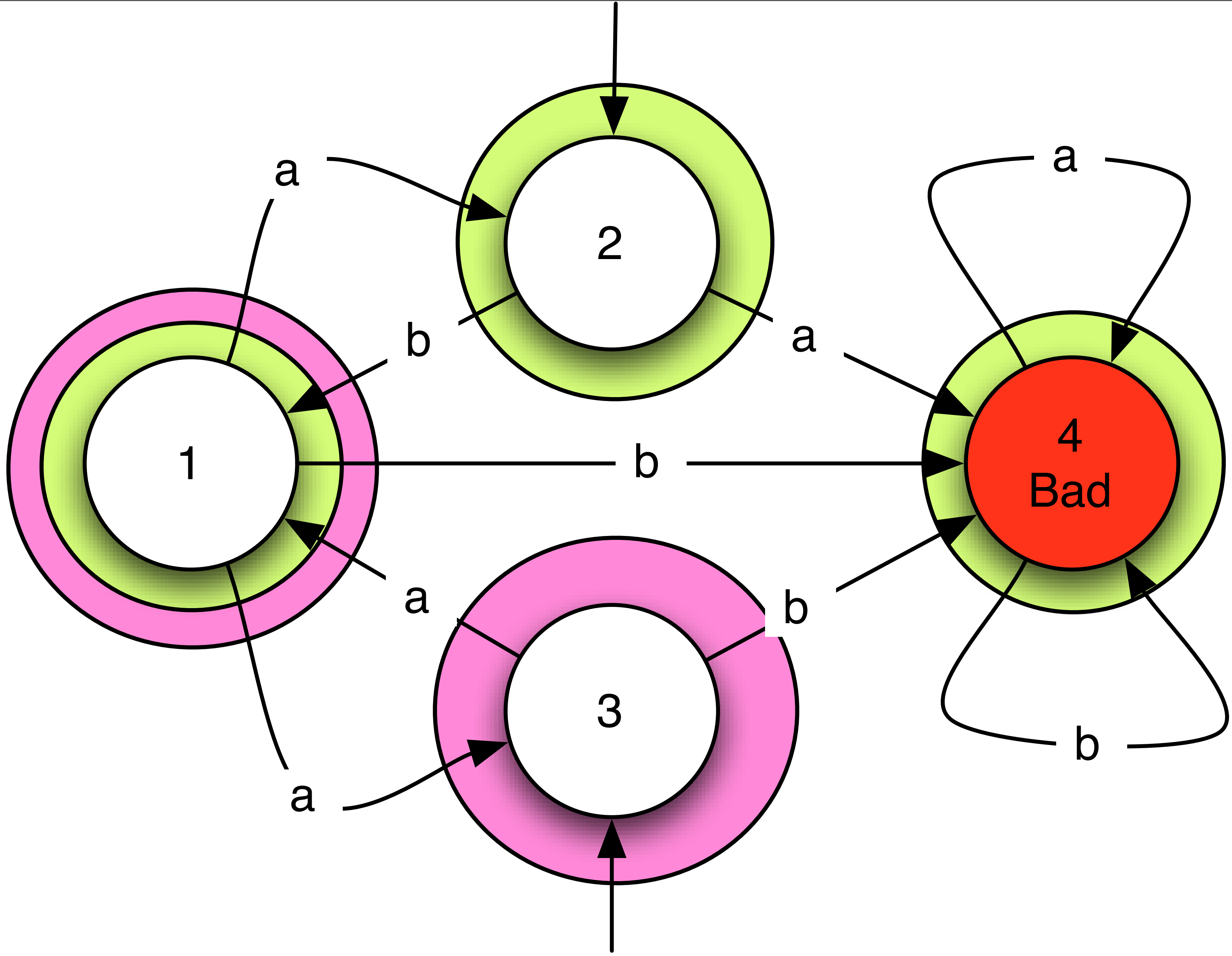
Indeed,

$\mathsf{Post}_a(\{1\}) \cap \{1,2,4\} \subseteq \{2\}$

$\mathsf{Post}_a(\{1\}) \cap \{1,3\} \subseteq \{3\}$

Adding any state would break this property



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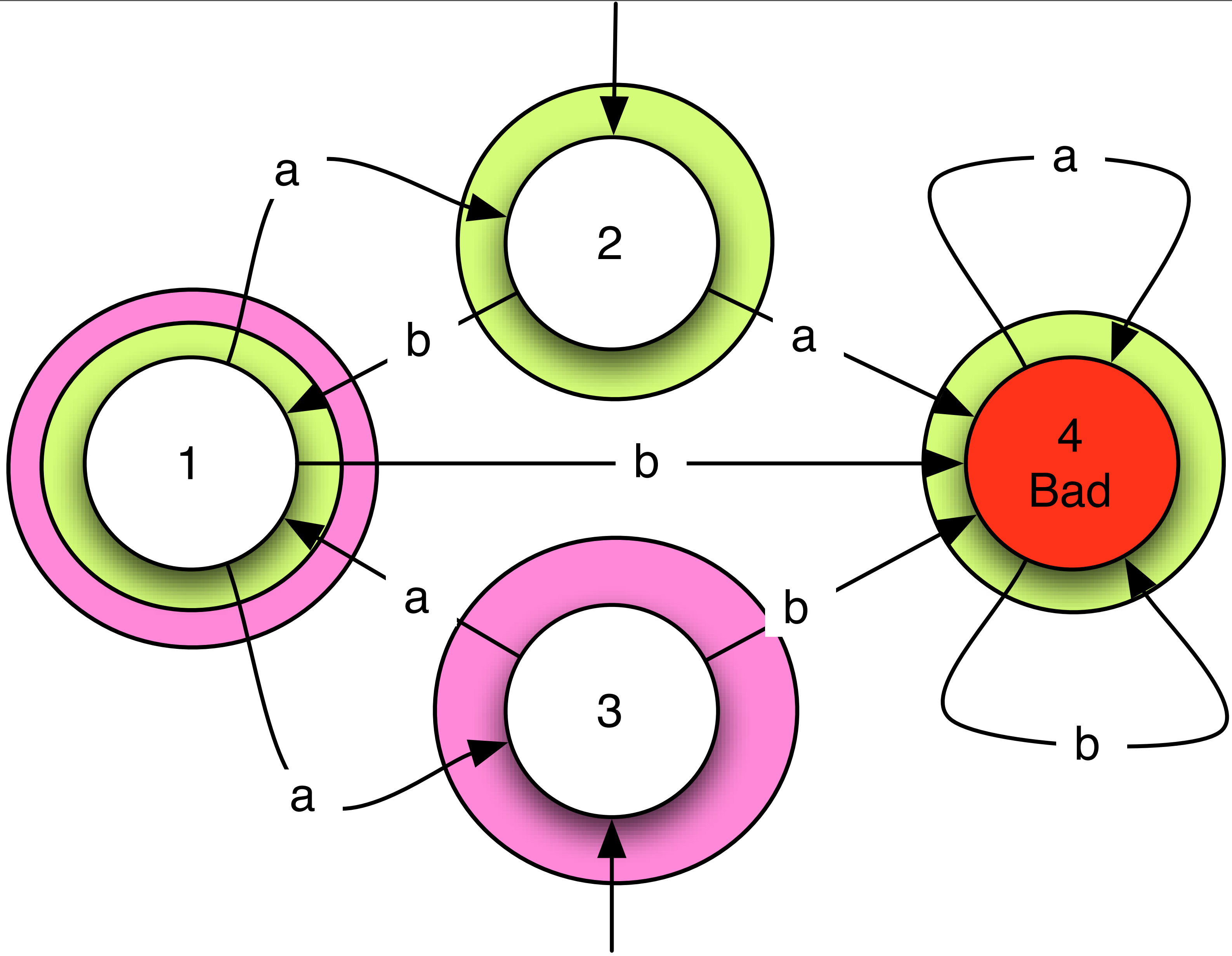
Fixed point



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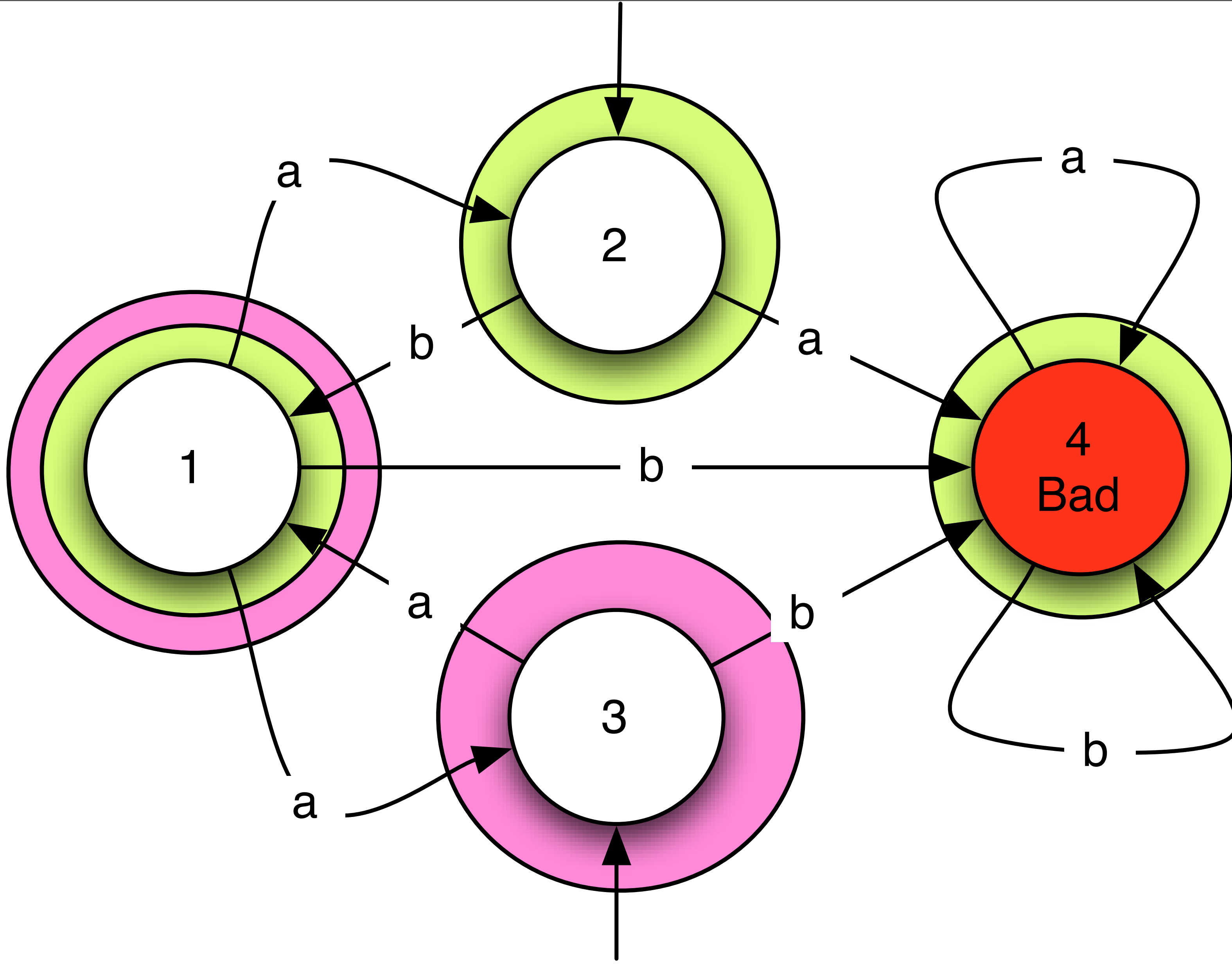
Fixed point

We have

$$\{\{2,3\} \cap \mathsf{Obs}_0, \{2,3\} \cap \mathsf{Obs}_1\} \sqsubseteq \sqcup\{q \mid q = \mathsf{CPre}(q)\}$$

and so, Player 0 has an observation based winning strategy to avoid Bad



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Fixed point

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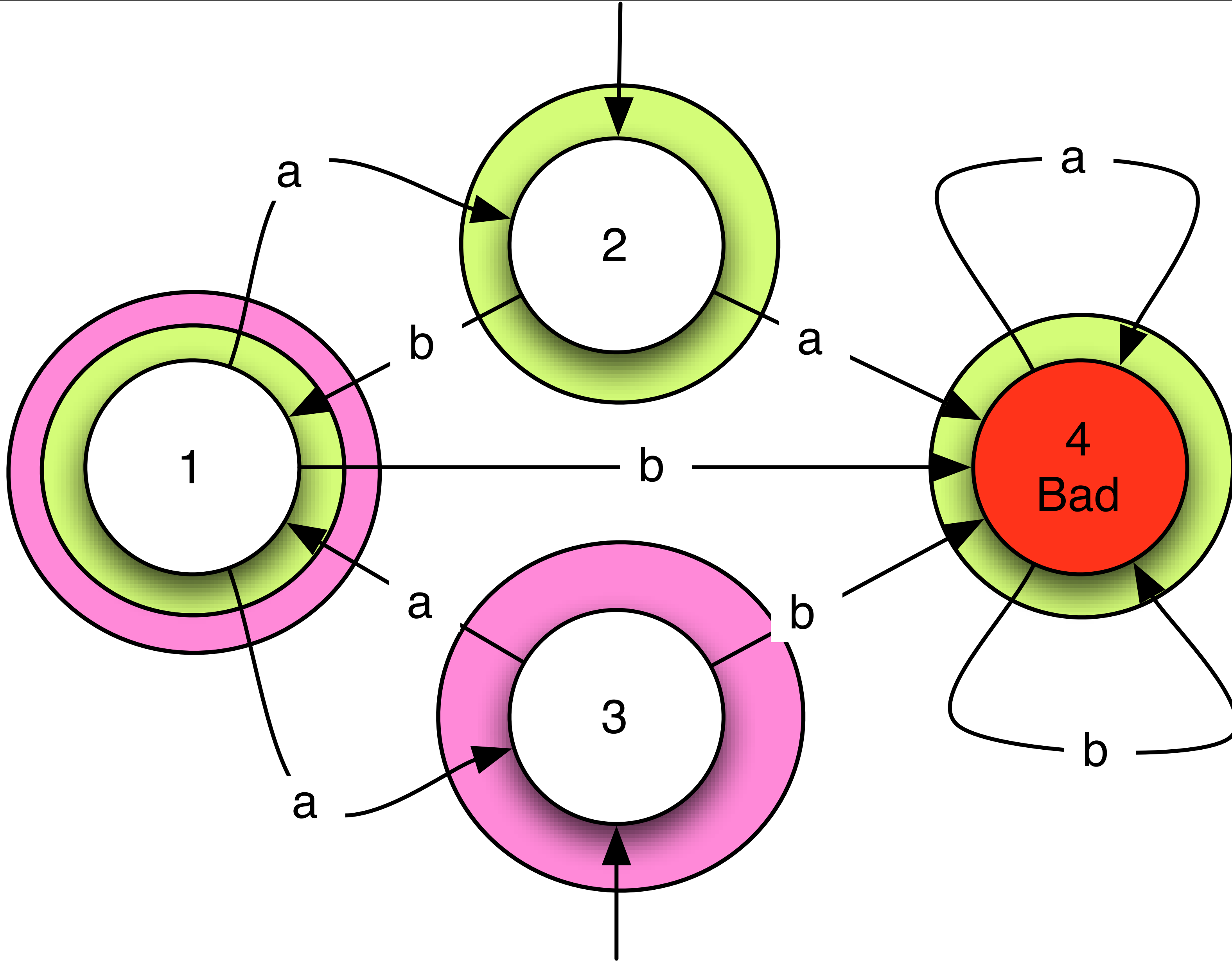

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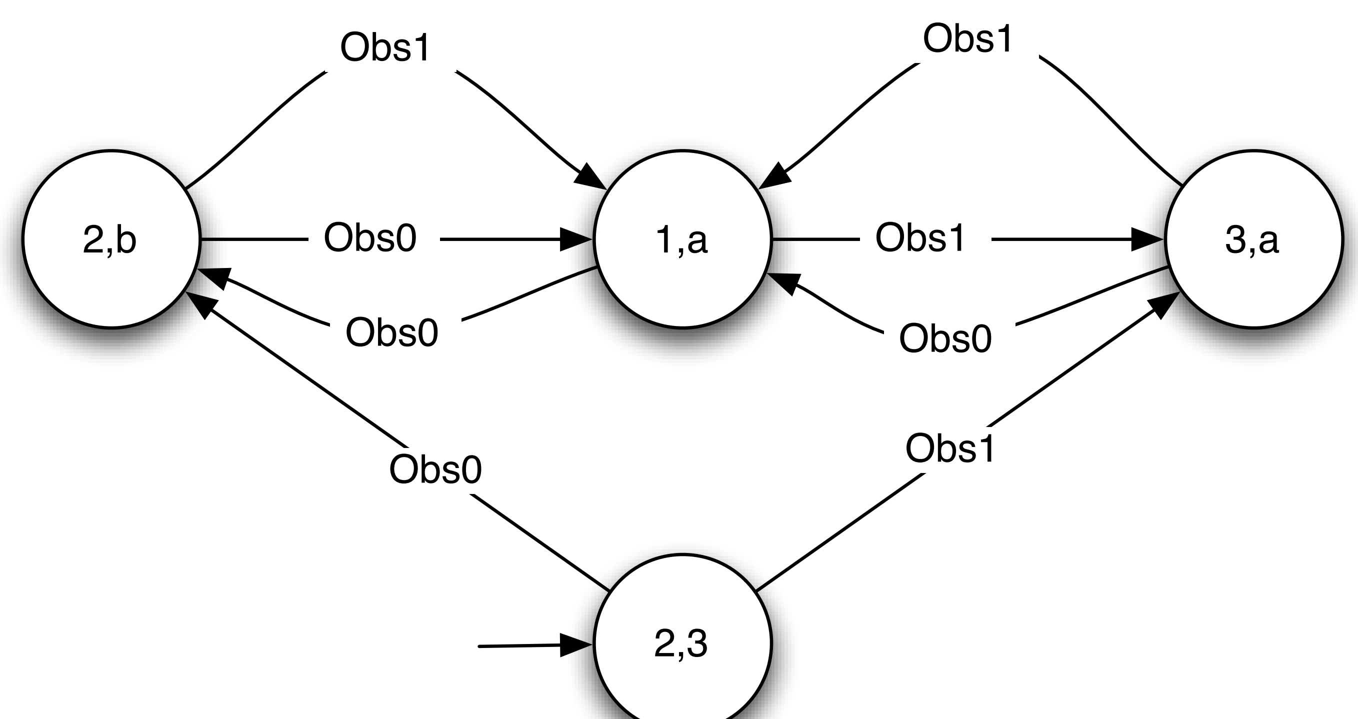
Fixed point



Complexity for finite state games

- The imperfect information control problem is *EXPTIME-complete*
- There exist finite state games of incomplete information for which the algorithm of [Rei84] requires an exponential time where our algorithm needs only polynomial time

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We compute exactly what is needed to control the system for a given objective

Infinite state games

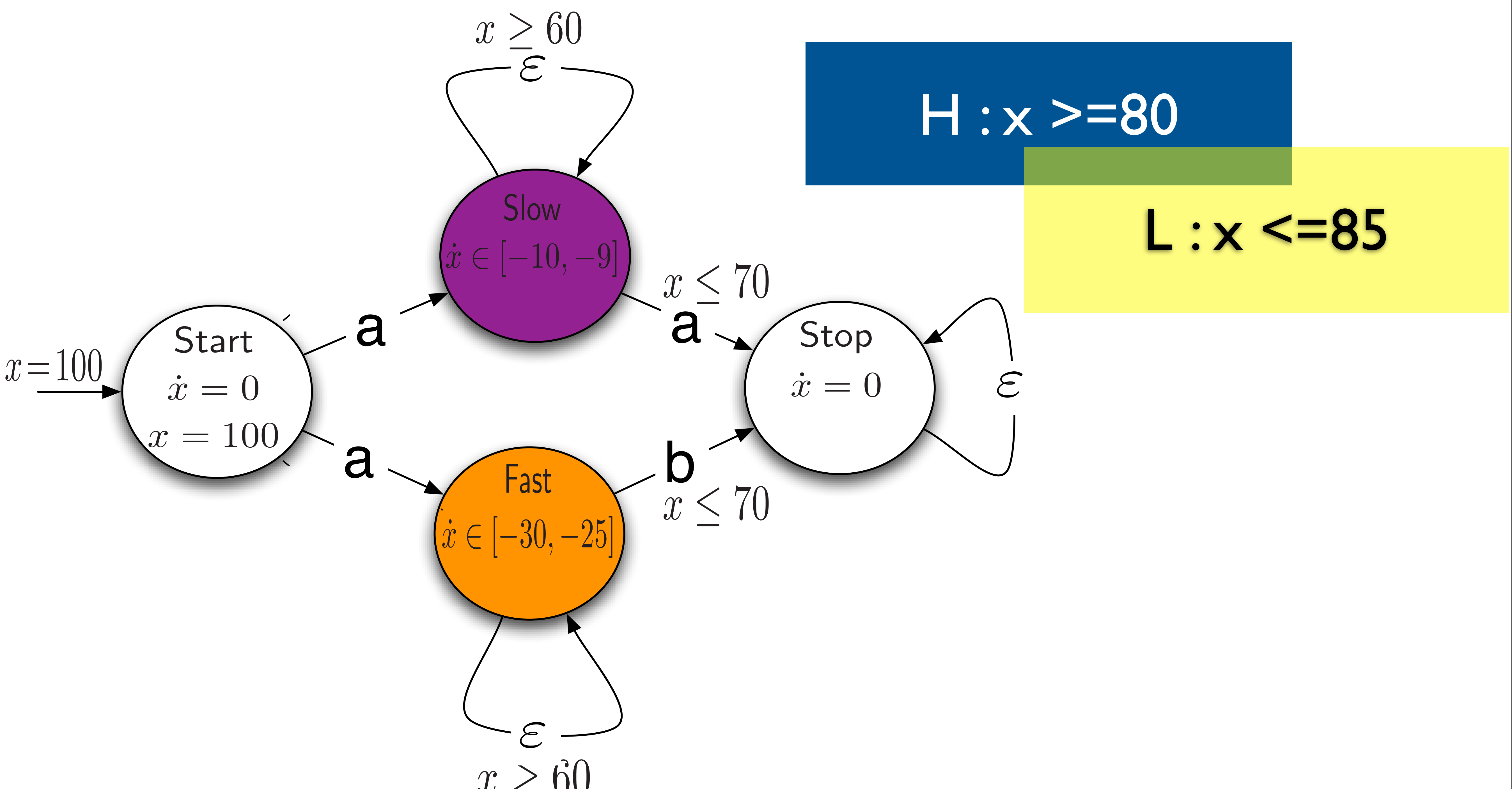
We can drop the assumption that S if finite

Our fixed point algorithm will terminate if

There exists a finite quotient of the state space in which Post, Enabled, γ are definable using this quotient

Application : Discrete Time Control of RHA

Discrete time control of RHA

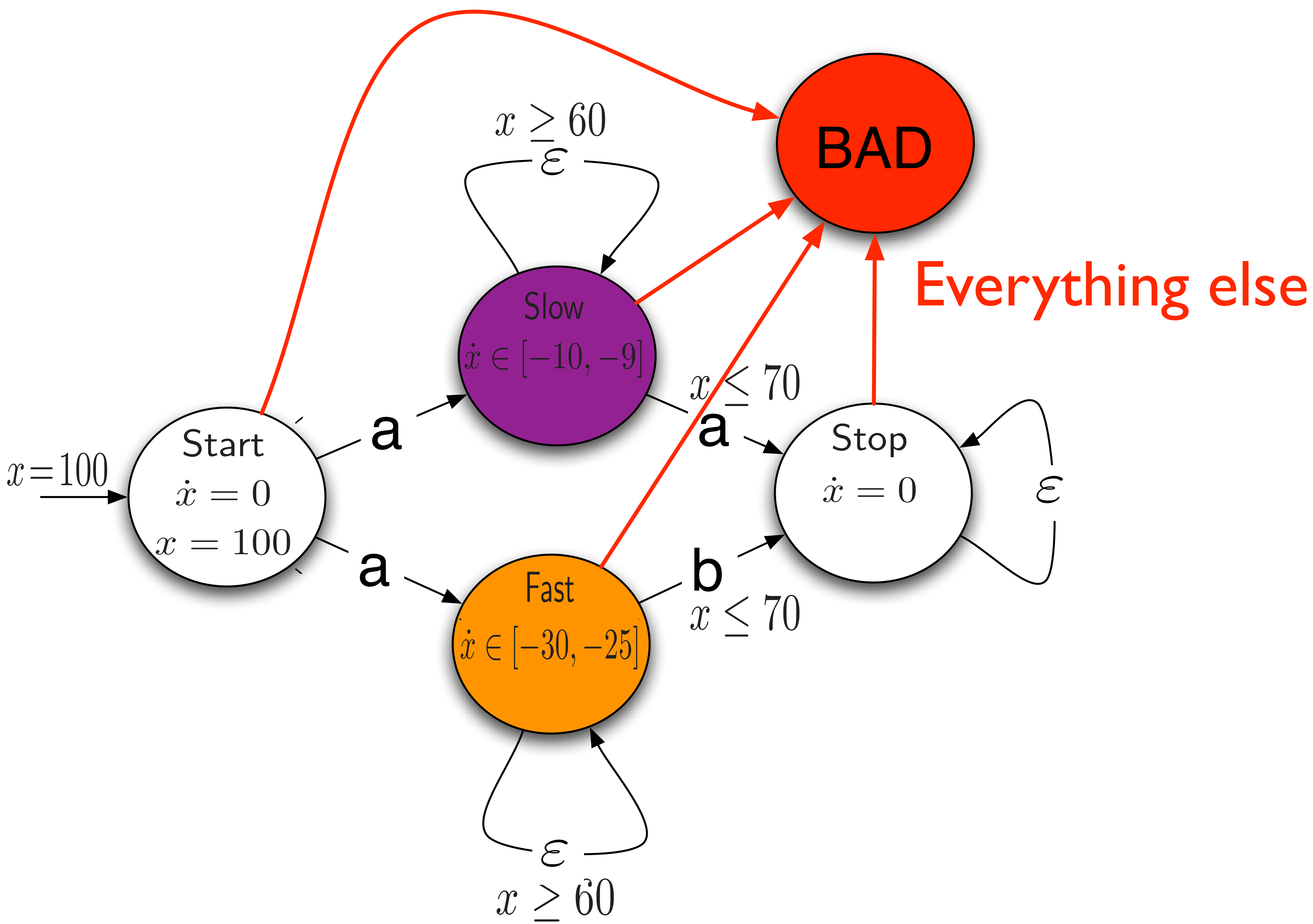


Player 1 (contr.) chooses an action every 1 time unit

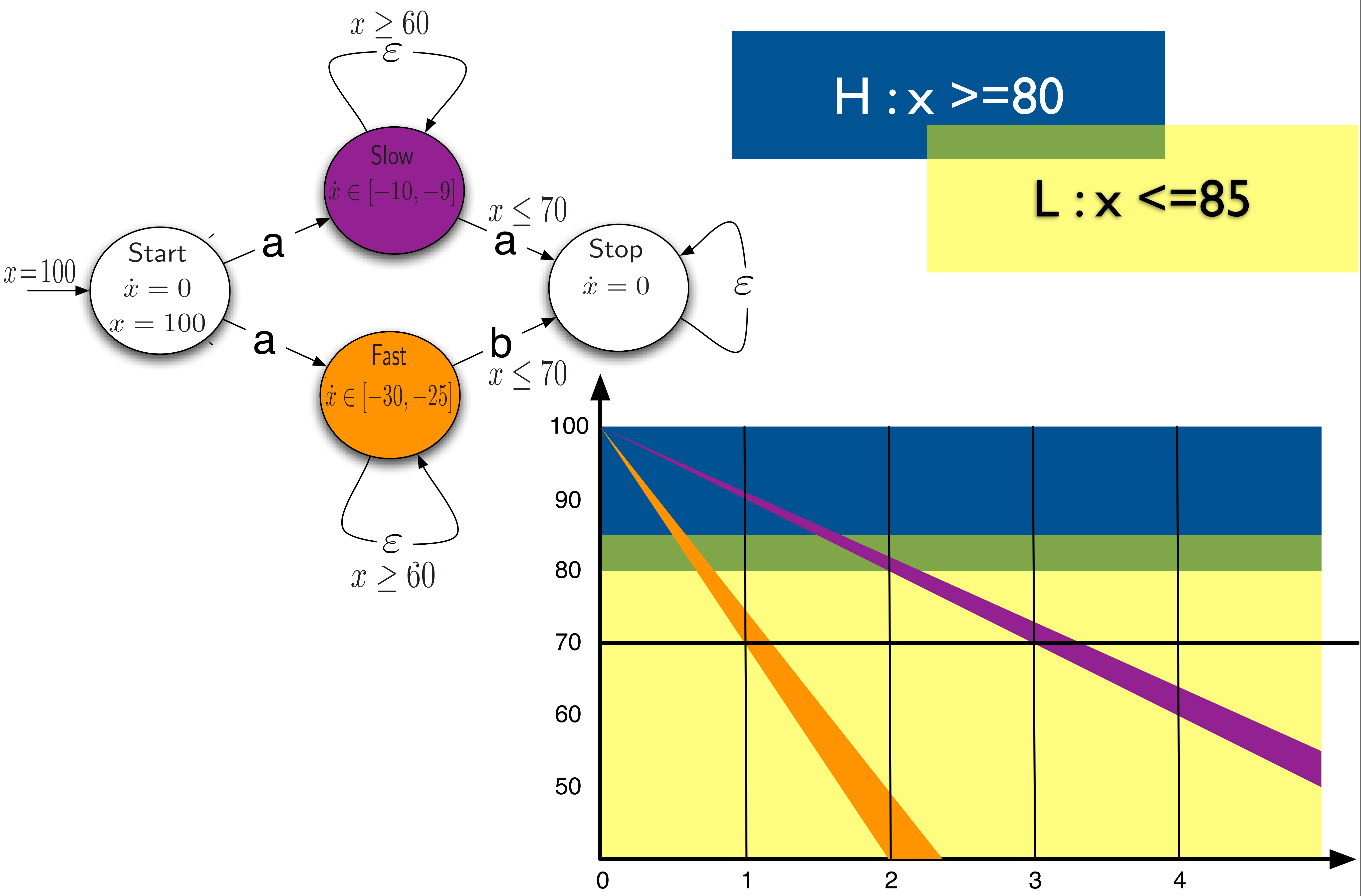
Player 2 (env.) resolves nondeterminism

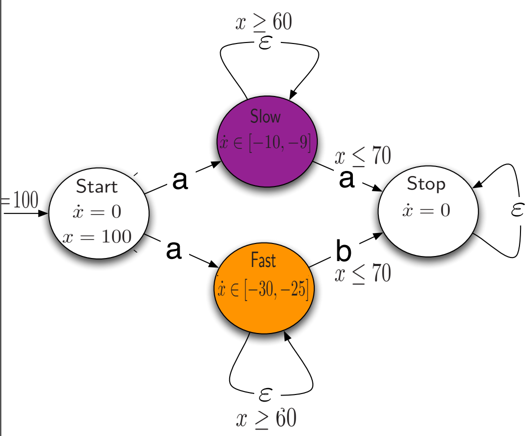
(in discrete and continuous steps).

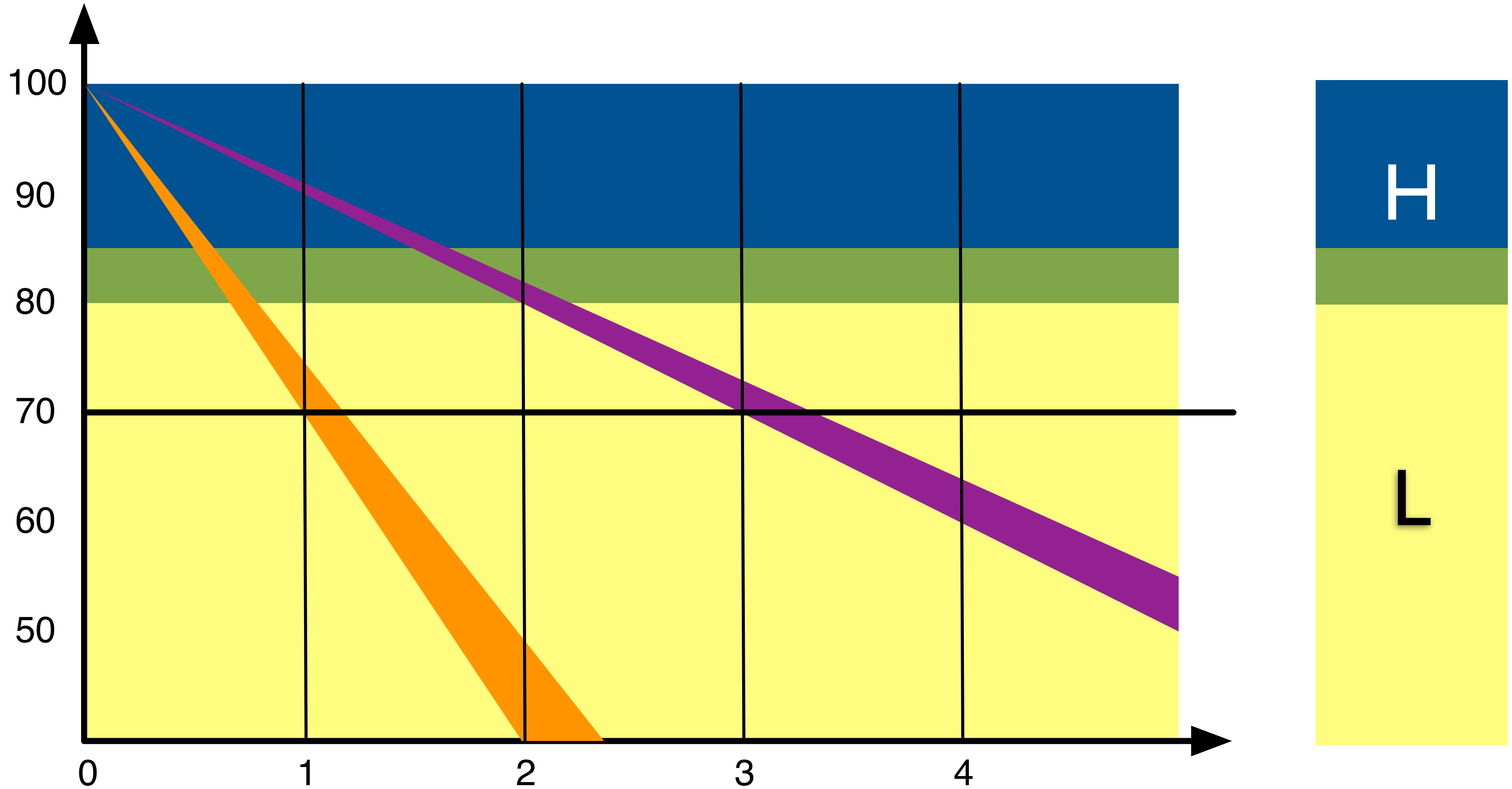
Discrete time control of RHA

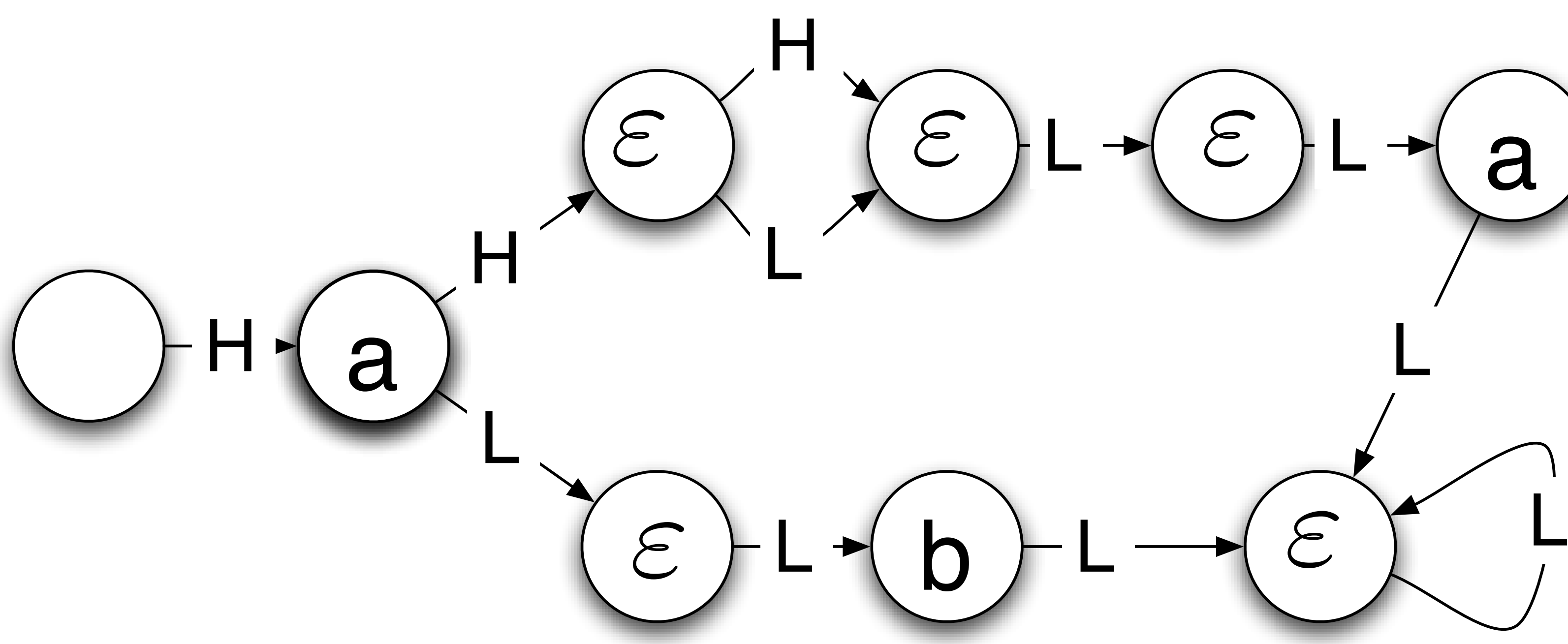


Discrete time control of RHA

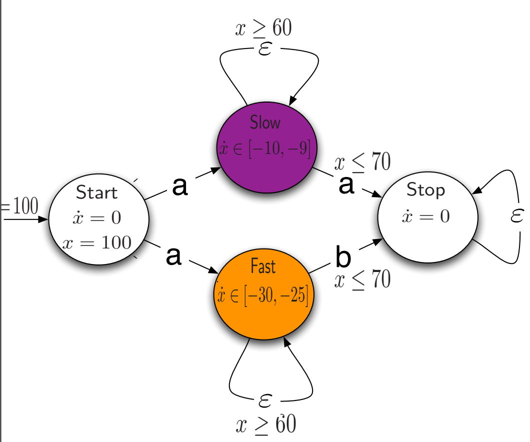


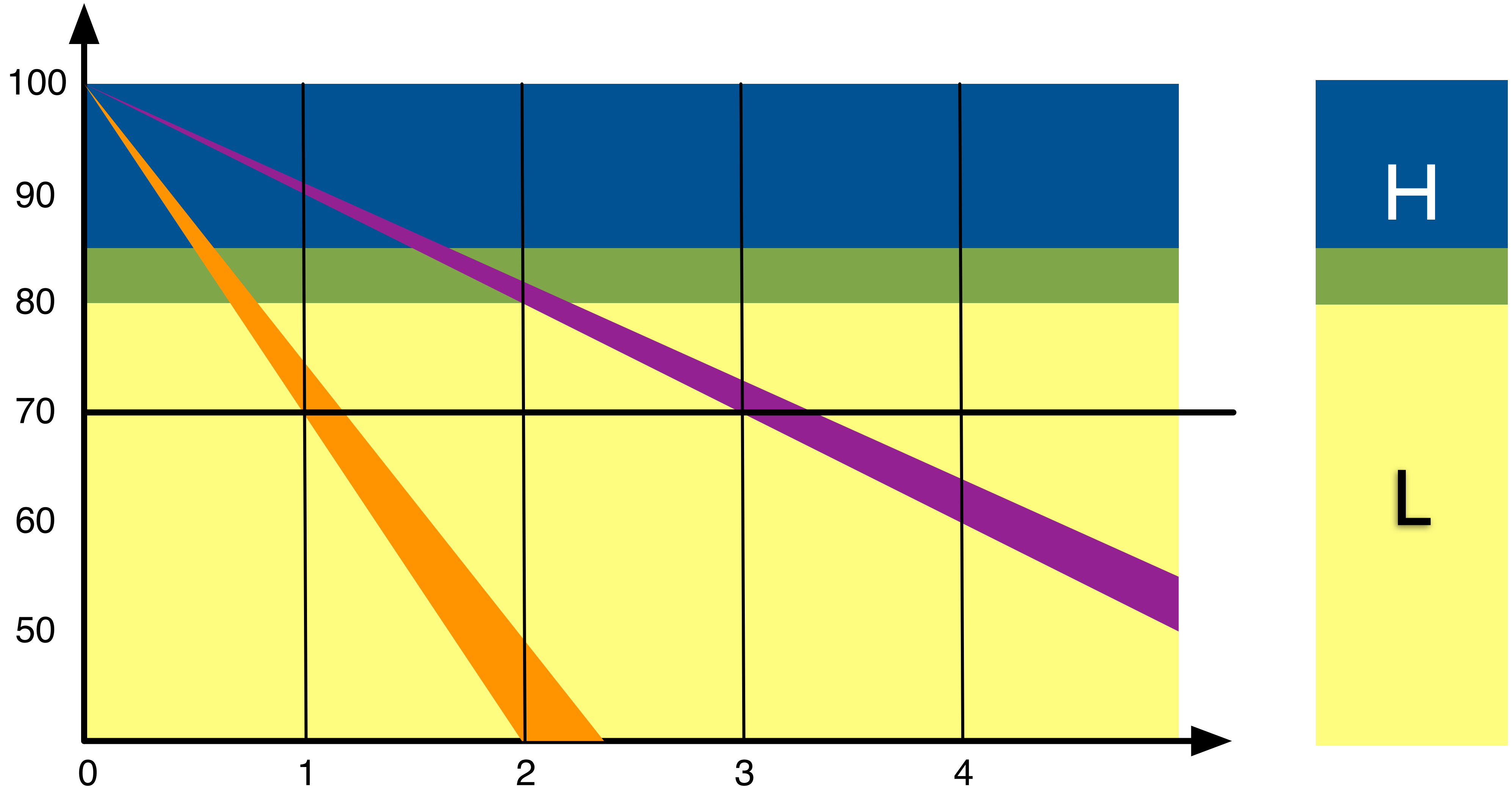


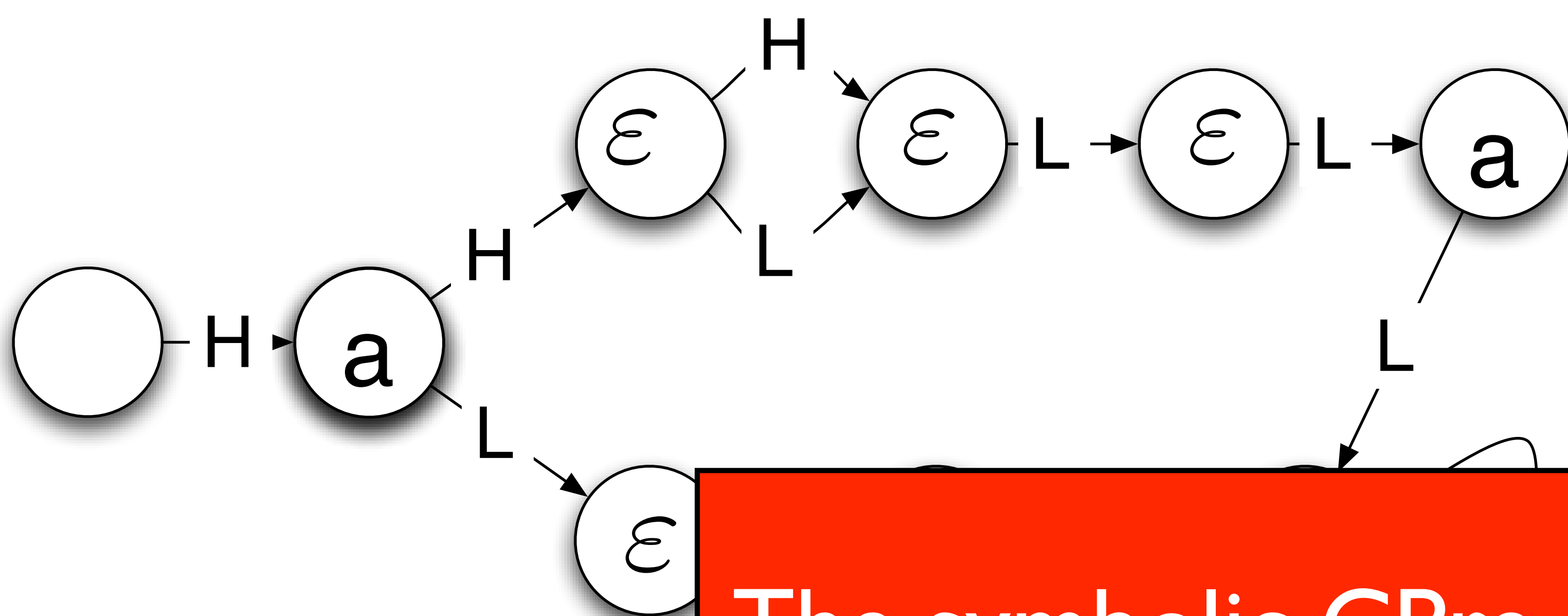




The Strategy

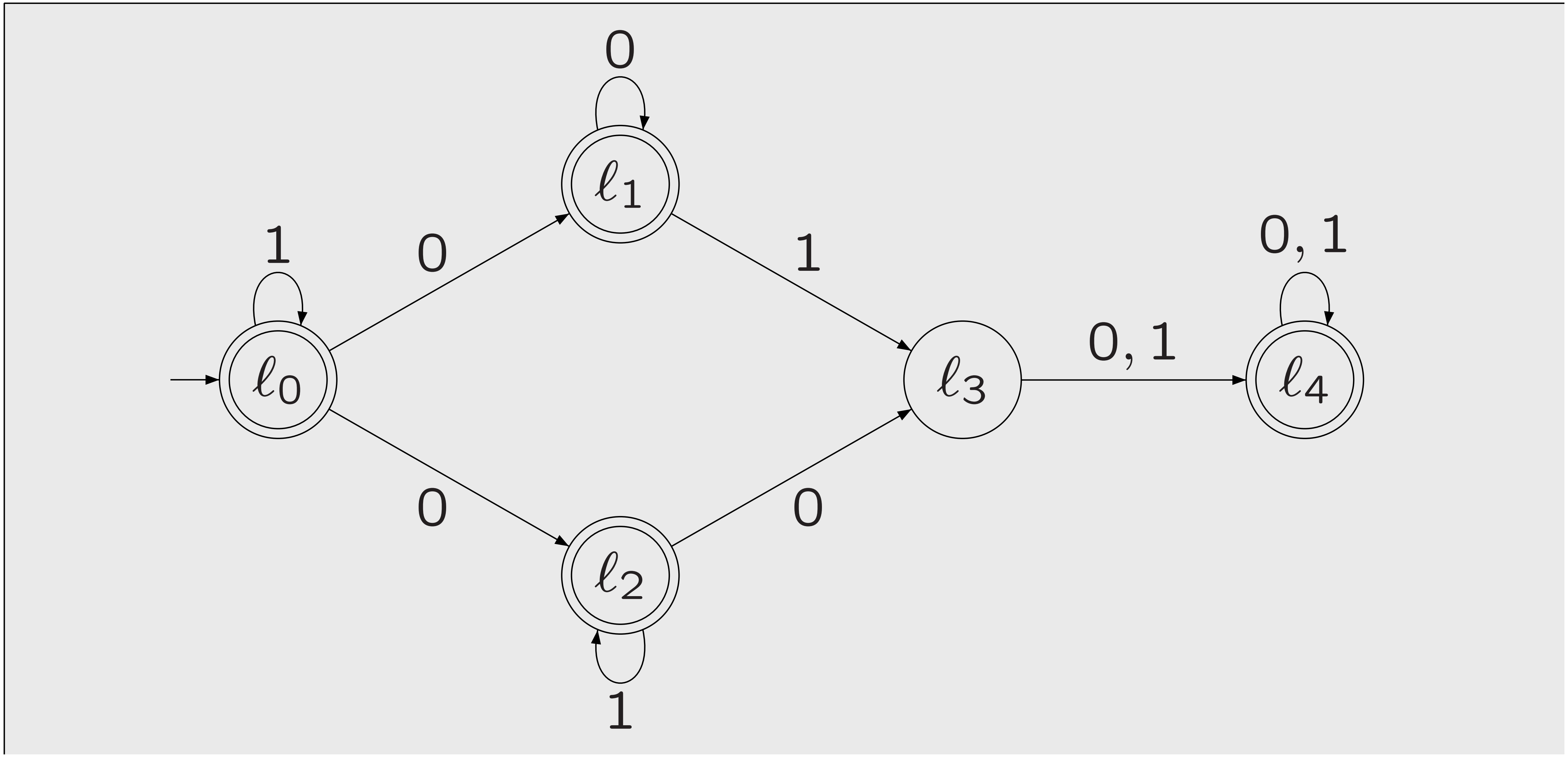






The symbolic CPre can be encoded in the script language of HyTech

Universality of NFA



Universality of NFA

Consider a game played by a protagonist and an antagonist

The protagonist wants to establish that $\mathcal{A}$ is not universal.

The protagonist has to provide a finite word w such that no matter how the antagonist reads it using $\mathcal{A}$, the automaton ends up in a rejecting location.

$\Longrightarrow$ This is a one-shot game.

Universality of NFA

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$\Longrightarrow$ This is a one-shot game.

The game is turn-based: the protagonist provides the word w one letter at a time, and the antagonist updates the state of $\mathcal{A}$. The protagonist cannot observe the state chosen by the antagonist.

$\Longrightarrow$ This is a blind game (or game of null information).

Let $\mathcal{A} = \langle \mathsf{Loc}, \ell_I, \Sigma, \delta_A, F \rangle$.

Consider the following controllable predecessor operator over sets of sets of locations. For $q \subseteq 2^{\mathsf{Loc}}$, let:

$$\boxed{\mathsf{CPre}(q) = \{s \mid \exists s' \in q \cdot \exists \sigma \in \Sigma \cdot \forall \ell \in s \cdot \forall \ell' \in \mathsf{Loc} : \delta_A(\ell, \sigma, \ell') \rightarrow \ell' \in s'\}}$$

So $s \in \mathsf{CPre}(q)$ if there is a set $s' \in q$ that is reached from any location in s, reading input letter σ, that is $\mathsf{Post}_\sigma(s) \subseteq s'$.

$\implies$ CPre encodes the blindness of the game.

Let $\mathcal{A} = \langle \mathsf{Loc}, \ell_I, \Sigma, \delta_A, F \rangle$.

Theorem:

$$\{\ell_I\} \in \mu x.(\mathsf{CPre}(x) \cup \{T\})$$

iff

Protagonist has a strategy to win G_T

iff

$\mathcal{A}$ is not universal

Claim: For $s_1 \subseteq s_2$, if $\mathsf{Post}_\sigma(s_2) \subseteq s'$ then $\mathsf{Post}_\sigma(s_1) \subseteq s'$ and if $s_2 \in \mathsf{CPre}(\cdot)$, then $s_1 \in \mathsf{CPre}(\cdot)$

Idea: Keep in $\mathsf{CPre}(x)$ only the maximal elements.

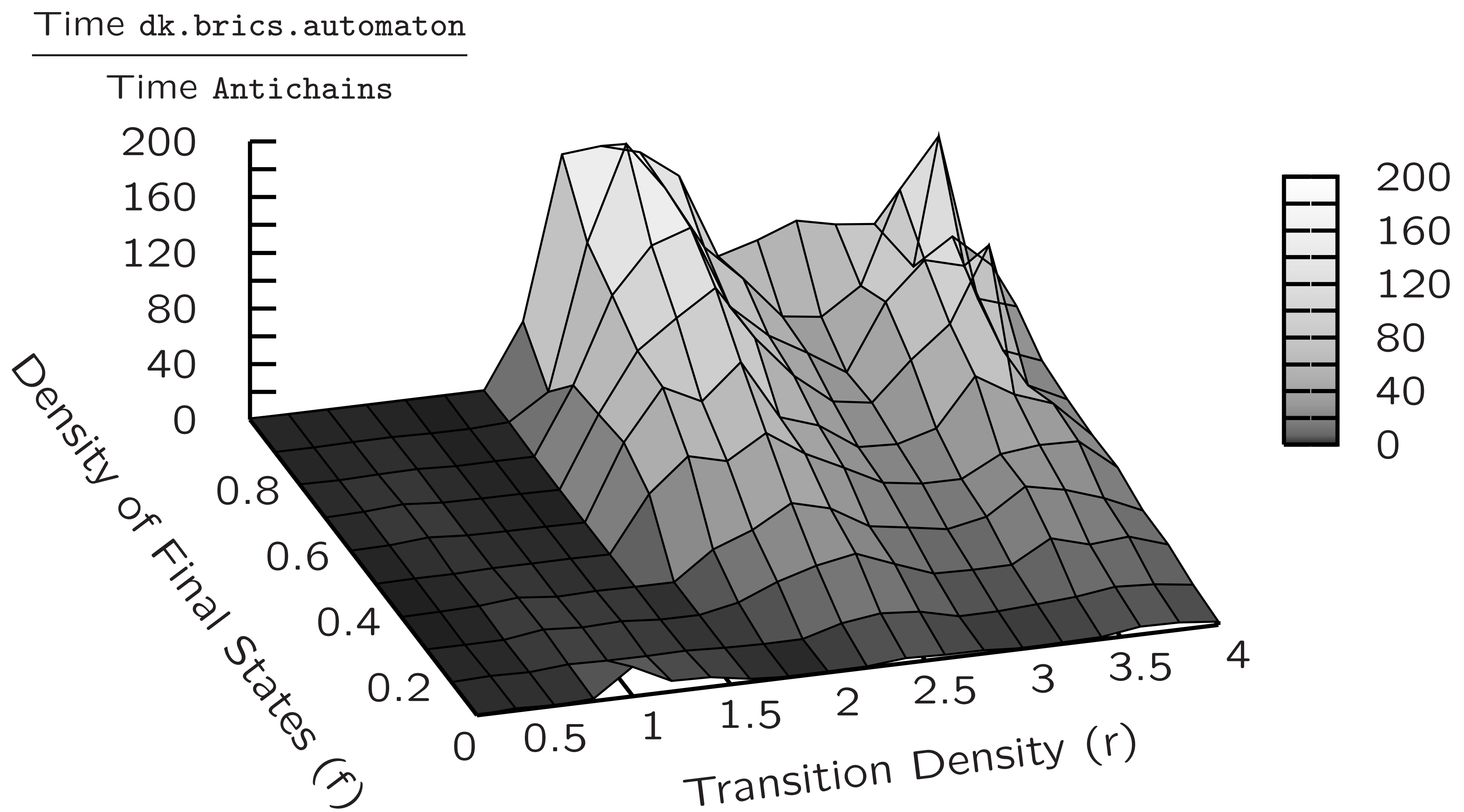
Universality - Experimental results (1)

- We compare our algorithm `Antichains` with the best[(1)] known algorithm `dk.brics.automaton` by Anders Møller.

  (1) According to "*D. Tabakov, M. Y. Vardi. Experimental Evaluation of Classical Automata Constructions. LPAR 2005*".

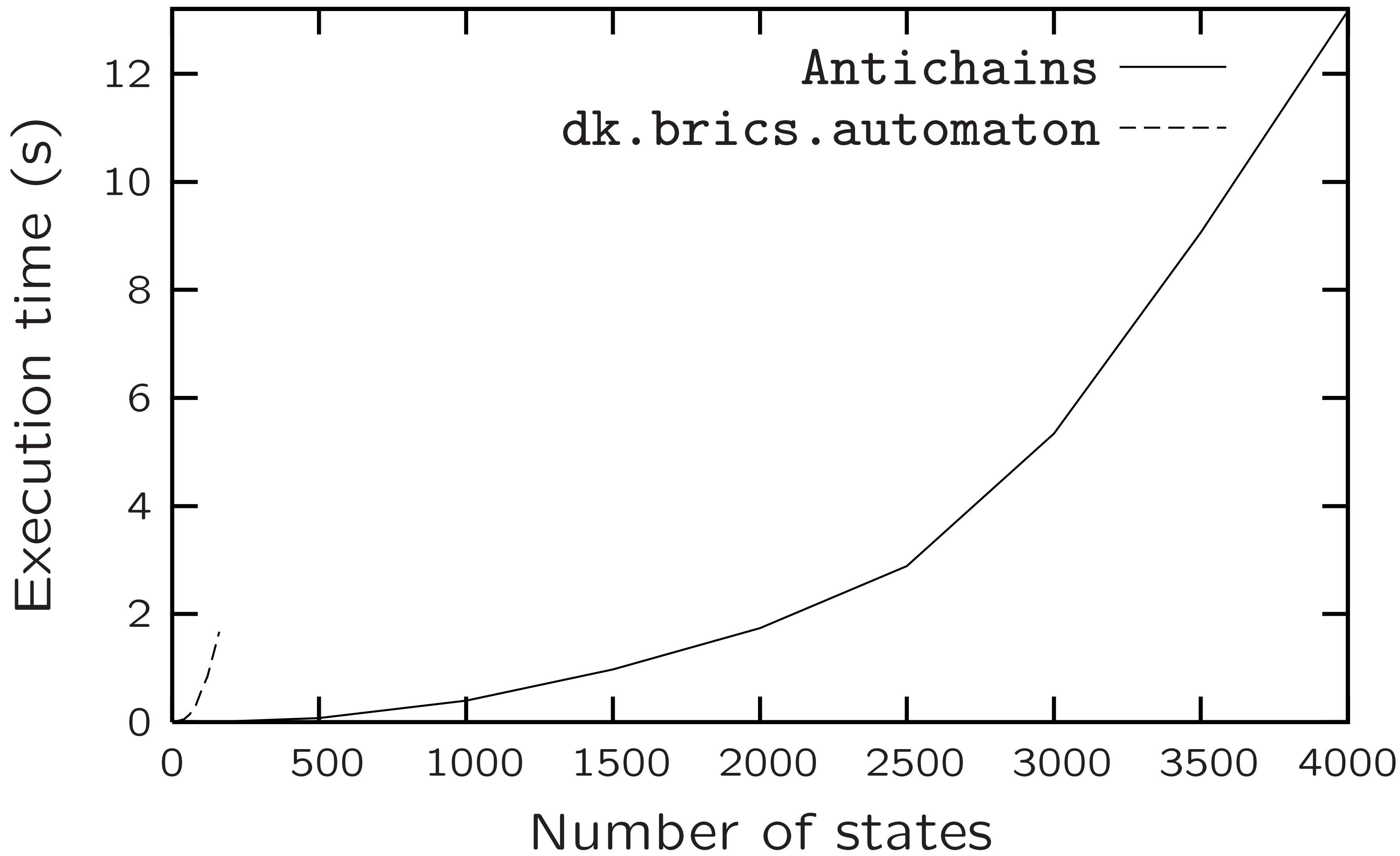
- We use a randomized model to generate the instances (automata of 175 locations). Two parameters:
  - Transition density: $r \geq 0$
  - Density of accepting states: $0 \leq f \leq 1$

Universality - Experimental results (2)



Each sample point: 100 automata with $|\mathsf{Loc}| = 175$, $\Sigma = \{0, 1\}$.

Universality - Experimental results (3)



- Transition density: $r = 2$.
- Density of accepting states: $f = 1$.

Works also for

- *language inclusion* between NFA
- *emptiness* of AFA

- See proceedings of next CAV !

  (joint work with Martin De Wulf, Laurent Doyen and Tom Henzinger)

Conclusion/Perspectives

- We propose a **lattice theory** to solve games of imperfect information, those games are needed to make the synthesis of **robust controllers** (= finite precision). (see HSCC06)
- Our technique computes only the information that is needed to find a winning strategy, i.e. we **avoid** the explicit subset construction. Works for any regular objective (see CSL06)
- Applicable to **discrete time control** (see HSCC06) of RHA and useful to solve efficiently **classical problems** for NFA and AFA (see CAV06) and for automata on infinite words (submitted for publication)
- Perspectives : continuous time control, finite automata on infinite words, efficient implementation issues, etc.

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